Dynamic bi-level optimal toll design approach for dynamic traffic networks

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Delft University of Technology, 2007

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Dedicated to my parents

Dragan and Olivera

Preface

This dissertation contains the results of the research carried out at the Transport and Planning Department of the Delft University of Technology. My work was a part of the research programme Multi-Disciplinary study - Pricing in Transport (MD-PIT) funded by The Netherlands Organization for Scientific Research (NWO) and Connekt (The innovation network for traffic and transport in The Netherlands). Many people have contributed to finish this dissertation after 4 years of work. I apologize in advance if I omit someone.

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Notation

The following list shows an overview of sets of elements, indices, variables and parameters used in this thesis.

Sets

A	set of links in the network;
N	set of nodes in the network;
$R \subseteq N$	subset of origin nodes;
$S \subseteq N$	subset of destination nodes;
P^{rs}	set of paths from origin node r to destination node s;
$Y \subseteq A$	set of links that can be tolled (the set of tollable links);
M	set of user classes;
T	studied period of time;
$K \subset T$	set of departure time intervals;

Indices

$a \in A$	link index;
$r \in R$	origin node index;
$s \in S$	destination node index;
$p \in P^{rs}$	path-index for each OD-pair;
$k \in K$	departure time index;
$t \in T$	time period index;
$m \in M$	user class index;
$t_m^{ent} \in T$	time period in which traveler m enters the network;
$t_m^{exit} \in T$	time period in which traveler m exits the network;

Link variables

$c_{am}(t)$	travel costs of link a when entering the link at time t [eur];
$\tau_a(t)$	travel time on link a when entering the link at time t [min];
$u_a(t)$	inflow on link a when entering the link at time t [veh];
$v_a(t)$	outflow on link a when entering the link at time t [veh];
$x_a(t)$	number of vehicles on link a when entering the link at time t [veh];

Path variables

$\tau_p^{rs}(k)$	actual travel time on path p for users departing from origin r to destination s in time interval k [min];
$q_{pm}^{rs}(k)$	path flow rate of travelers class m departing from origin r to destination s in time interval k along path p [veh/min];
$\pi_m^{rs}(k)$	minimal travel cost for class m users departing during time interval k from origin r to destination s [eur];
$c_{pm}^{rs}(k)$	actual route travel cost for traveler class m departing during time interval k from origin r to destination s along route p [eur];
$U_{pm}^{rs}(k)$	total utility for traveler user class m using path p starting in departure time k between origin r and destination s;
$V_{pm}^{rs}(k)$	systematic utility for traveler class m using path p at departure time k between origin r and destination s;
$\varepsilon_{pm}^{rs}(k)$	unobserved utility for traveler class m using path p at departure time k between origin r and destination s;
$\overline{V}_m^{rs}$	utility of spending time at destination s departing from origin r for user class m;
τ_m	travel time of traveler class m;
L	the length of the trip;
D	the duration of the trip;

Demand variables

D_m^{rs}	total travel demand between origin-destination pair (rs) for user class m [veh/h];
$D_m^{rs}(k)$	travel demand between origin destination pair (rs) in time interval k for user class m [veh/h];

Link-path variables

$\delta_{pam}^{rs}(k,t)$	the dynamic path-link incidence indicator for user class m departing in period k whether link a during period t is part of path p from r to s;
$u_{ap}^{rs}(k,t)$	inflows of link a at time interval t of vehicles traveling on route p from r to s;
$v_{ap}^{rs}(k,t)$	outflows of link a at time interval t of vehicles traveling on route p from r to s;

Toll variables

$\theta_{pm}^{rs}(k)$	total toll on path p when departing during time interval k from r to s for user class m [eur];
$\theta_{am}^{\min}$	minimum toll value on link a (for all time intervals) for traveler class m [eur/passage];
$\theta_{am}^{\max}$	maximum toll value on link a (for all time intervals) for user class m [eur/passage];
$\theta_{am}^{\min}(t)$	minimum toll value on link a at time period t for user class m [eur/passage];
$\theta_{am}^{\max}(t)$	maximum toll value on link a at time period t for user class m [eur/passage];
$\theta_{am}(t)$	toll on link a when entering the link a at time period t for user class m [eur/passage];
$\theta(s,t)$	the fare charged at locations s during time period t [eur/passage];
$\theta_a(t)$	variable fare [eur/passage];
$\overline{\theta}_a$	single maximum fare value [eur/passage];
$\phi(t)$	given proportions of fare value;

Parameters

μ	scale parameter of the utilities in the joint logit model at the path and departure time choice level;
PDT^{rs}	preferred departure time interval for travelers from origin r to destination s;
PAT^{rs}	preferred arrival time interval for travelers from origin r to destination s;
α_m	value of time for user class m [eur/min];
β_m	penalty for deviating from PDT for user class m [eur/min];
γ_m	penalty for deviating from PAT for users class m [eur/min];

Game theory notation

c_{ip}	generalized path cost function of traveler i using path p [eur];
α_i	value of time of traveler i [eur/h];
τ_p	travel time of path p [h];
θ_p	toll cost on path p [eur];
U_{ip}	trip utility of traveler i for making a trip using path p [eur];
$\overline{U}$	utility for making a trip [eur];
$\overline{S}_i$	set of available travel strategies of traveler i;
$\overline{\Theta}$	set of available toll strategies of the road authority;
$s_i \in \overline{S}_i$	possible travel strategy of traveler i;
s_i^*	optimal travel strategy (path) of traveler i;
$s_{-i} \in \overline{S}_i$	travel strategies for all other travelers;
s_{-i}^*	chosen travel strategies of all other travelers;
$\theta \in \overline{\Theta}$	possible toll strategy of the road authority;
θ^*	optimal toll strategy (tolls) of the road authority;
J_i	utility payoff of traveler i [eur];
$\overline{R}$	utility payoff of the road authority [eur];
q_p	number of travelers using path p.

Acronyms

MPEC	Mathematical Program with Equilibrium Constraints
DTA	Dynamic Traffic Assignment
DOTD	Dynamic Optimal Toll Design problem
DNL	Dynamic Network Loading
DUE	Deterministic User Equilibrium
DDUE	Deterministic Dynamic User Equilibrium
MSA	Method of Successive Averages
MNL	MultiNomial Logit
SUE	Stochastic User Equilibrium
PS	Path Size
SDTA	Stochastic Dynamic Traffic Assignment
DSUE	Dynamic Stochastic User Equilibrium
NDP	Network Design Problem
BLP	Bi-Level Program
VIP	Variational Inequality Problem
VOT	Value Of Time
VOSD	Value Of Schedule Delay

Part I

Optimal toll design problem specification

Chapter 1

Introduction

1.1 Pricing as a policy instrument in transport planning

Direct pricing of trips, for example using tolls, is widely advocated to solve problems in transportation planning such as congestion, environmental impacts, safety and the like. Pricing of trips is not a really new policy instrument. It has for example a very successful history in controlling parking in inner cities all over the world.

In many countries some form of road pricing is already functioning well, be it as a means to control the level of demand for car trips, to regulate the use of scarce capacity during peak hours, or to charge the road users for the cost of using new infrastructure (e.g. congestion charging in London, revenue generation in Spain and France, pricing in Singapore, toll roads in Italy).

With road pricing we define the charging of the road user for using a particular part of the road network during conducting the trip. The money to be paid is called the toll.

Since already very long, road pricing is proposed by economists as an instrument to make the transport system more efficient in the sense that by this means external effects of individual traveler's road usage may be internalized, thus forcing travelers to make more efficient travel decisions from a welfare-economic point of view (see e.g. Walters (1961), Verhoef et al. (1999)).

In this thesis we will not follow this line of motivating and designing road pricing.

Instead, we will take the position of a road authority trying to improve the performance of the transport system for which it is responsible, by adopting some form of road pricing. This performance may for example relate to the traffic operations in the system (congestion, delays, reliability, throughput, etc.), to its traffic impacts (on safety, or the environment), or to the costs of the system (cost recovery objective of road pricing). To achieve its goals, the road authority will formulate specific performance objectives to be attained (for example reducing the level of congestion by at least 30%) and may specify how, with which instrument or package of instruments, it is proposing to solve his objective most effectively. For an overview, see Verhoef et al. (2004), Verhoef & Small (2004). It should be noted that attaining the policy objective should not be considered as an isolated goal of the road authority. However, in the case of congestion reduction, imposing high travel costs can reduce congestion but the society will suffer. Therefore, the acceptability of proposed instruments or measure play an important role. For more information about acceptability of road pricing see Kalmanje & Kockelman (2004), Verhoef et al. (2004), Steg et al. (2006), Ubbels (2006).

In order to assess the effectiveness of such policy plans with respect to road pricing, quantitative analysis tools (models) are needed that may predict the likely impacts of introducing particular forms of road pricing as a means to solve particular problems as expressed in the authority's objectives. Such modeling tools for analyzing given pre-specified tolling plans we call a toll impact model. For an overview, see Yang & Huang (2005).

Apart from such tools that are able to predict the likely consequences of proposed tolling plans specified in advance by the road authority, one may think one step further of quantitative tools (models) that may derive the best tolling pattern to be applied given a specific planning objective of the road authority. We call this type of modeling tool a toll design model, since it is able to determine the optimal combination of characteristics of a toll regime, consisting of where, when, from whom and how much toll to levy. The necessity of such a pricing design tool follows from the enormous complexity of designing an effective tolling system in practice, even if only a few toll locations are involved. Apart from the huge dimensionality of the design task following from the potential numbers of toll locations, toll periods, toll levels, and traveler types to be tolled, the more difficult aspect of the design problem is in the multitude of potential behavioral responses of the travelers to the incurred tolls such as for example shifts in trip frequency, route choice, departure time choice, mode choice etc. For an overview, see Verhoef et al. (2004).

In this thesis we will take up the challenge and will develop a toll system design tool. We use the term 'toll system' since at least three dimensions are taken into account in the design, namely the locations, the periods, and the levels of the tolls to be levied throughout the network. The modeling tool will be able to specify the optimal design of a toll system as the answer to a specific performance objective of the road authority and will give the corresponding performance characteristics of the tolled transport system.

In developing this tool we will specifically take a number of specific conditions into account:

- the tool should be able to correctly address dynamic networks, meaning that travel demand, network flows, travel times, capacities and the like may vary over time;

- the tool should be able to handle correctly the heterogeneous composition of the travel demand (so called multi-class flow) with respect to drivers to be tolled differently and with respect to different behavioral responses to the pricing;

- the toll levels should be dynamic in the sense of time varying or maybe even flow-level varying.

Addressing these specific conditions forms the outstanding characteristic of this toll design model.

In the following sections, we will elaborate on the subject of this thesis, ending up with a formulation of the modeling task to be solved.

We will first sketch the research context of this thesis project as being part of a larger multi-disciplinary research program on road pricing. Then we will address the planning context of our endeavor specifying in somewhat more detail the roles of authorities and analysts, and specifying the type of policy objectives and policy instruments we like to include in our work. This includes a description of those properties of the transport system that are paramount in a tolling design study.

Given this, we will specify our research questions that will be addressed in the thesis, followed by our account of the scientific and practical contributions that our thesis is aimed to produce.

At the end this introductory chapter will describe the set up of the thesis.

1.2 Research context of the thesis

The policy issue of road pricing typically is a problem type concerning very many different aspects of daily life, not only traveling as such. While tolling mostly in the first place aims at influencing driver behavior, it has at the same time multiple other impacts. Because travel costs will change, travelers may decide to adapt their home or work locations in order to reduce the increased household expenditures. Equally, firms may reconsider their current locations in order to prevent their employees and clients from increased transportation costs. Such processes imply that road pricing may lead to shifts in spatial distribution patterns of households and firms which in turn may lead to shifts in spatial travel patterns. It is possible that employers may reimburse the money spent on tolls to employees (having as a consequence change in salaries). Another important aspect of road pricing are the induced money streams of toll revenues: how will these revenues be used, for what purposes, and with what potential impacts? Since a net positive revenue is not secured at all because of the high investment and operational costs of such systems, a serious question in each particular toll system proposal concerns the financial viability of the proposal. Another relevant policy concern is the acceptance of some form of road pricing by the general public: in the public there maybe conflicting opinions about the use of the revenues, the social equity of the tolling measures, the privacy of the tolling data, the effectiveness of the proposed measures, etc.

In order to shed more light upon these interrelated road pricing issues, a consortium of Dutch universities has launched in 2002 a multidisciplinary research program called MD-PIT: Multi-Disciplinary Study of Pricing in Transport. This ongoing program is being

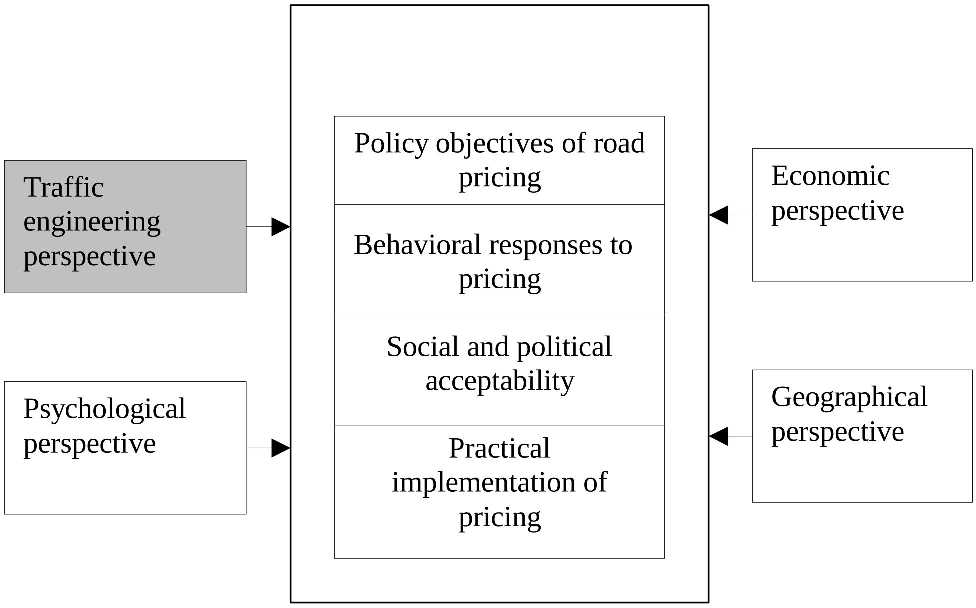

Figure 1.1: Road pricing from different perspectives (from MD-PIT project)

sponsored by the Transportation program (VEV) of the Dutch National Science Foundation NWO.

The purpose of this program is to study a variety of road pricing issues from various relevant perspectives such as the traffic engineering perspective, the economic perspective (see Ubbels (2006)), the psychological perspective, and the geographical perspective (Tillema (2007)). Main topics to be studied are the role of various policy objectives, behavioral responses to pricing, the social and political acceptability of various pricing forms, and the practical implementation of pricing (see Figure 1.1).

This thesis is part of the traffic engineering subprogram of MD-PIT. For details about the other subprograms and projects of MD-PIT we refer to Tillema (2007), Ubbels (2006) and Steg et al. (2006) and sources given therein.

In the traffic engineering part of MD-PIT, three major streams of studies have been performed in cooperation with the other involved disciplines:

1. conducting a large scale survey aimed at collecting stated preference data on travel decision making by individual travelers in response to road pricing measures, and deriving from these responses a set of crucial parameters describing the choice behavior of travelers in case of road pricing such as price sensitivity, value of travel time gain/loss, value of schedule delays, value of travel time reliability, and the like;

2. based on the collected data ad (a), the development of a set of travel choice models (trip choice, route choice, departure time choice, etc.) for use in a comprehensive dynamic network flow prediction tool suitable for analyzing travel demand and traffic flow impacts of road pricing proposals;

3. developing and applying a design tool (including the models developed ad (b) for optimizing the system set-up (locations, periods, levels of the tolls to be levied, etc.) of road pricing regimes in dynamic networks.

This thesis reports on the achievements derived in part (3). For a detailed account of the results from parts (1) and (2), see for example Van Amelsfort et al. (2005a), Van Amelsfort et al. (2005b).

Apart from the developed design tool as such, the contribution of part (3) to the MD-PIT program consists of giving insights into the likely consequences of different types of tolling regimes and of different types of policy objectives pursued by road authorities. Specifically (see also Section 1.1), this pricing system design tool aims at application of dynamic tolls in dynamic networks considering a variety of user classes in the transport system. Consideration of the dynamics in travel demand and flow propagation in response to pricing is the distinguishing characteristic compared to the other MD-PIT studies (Tillema (2007), Ubbels (2006)).

1.3 Planning context of a toll system design tool

This thesis takes as a point of departure that there is a single road authority responsible for the provision of adequate transport infrastructure (road network) in a particular area as well as for the provision of adequate travel conditions in that network. For simplicity reasons we restrict our analyses in this thesis to the car vehicle network as such, thereby disregarding the links of such a network with the other parts of the transport network (such as for example the public transport system) and with the spatial system.

Interventions in the system such as road pricing needed to keep the system adequate are guided by a set of policy objectives of the authority with respect to the performance of its transport network. The policy objectives may relate to a large variety of issues such as for example:

- quality of traffic operations such as with respect to congestion levels, travel time delays, travel time reliability, throughput, etc.;
- acceptability of traffic impacts such as safety, environmental burdens (noise, emissions, fuel consumption, etc.), congestion externalities, etc.;
- cost recovery of road investments and maintenance;
- welfare: the contribution of the transport system to the society's economy and welfare at large.

Of course, introduction of road pricing might also be motivated by policy objectives that show no relationship at all to the functioning of the transport system (for example using revenues from road tolling for improving the state's or city's budget) or to that of the road network (for example collecting money from car users to improve public transport). In this thesis we focus on the type of objectives that concern the quality of traffic operations such as minimizing the systems total travel time, or the reduction of existing congestion levels by 50%, or improvement of travel time reliability in the system.

In order to achieve its objectives, the road authority has at its disposal a set of policy instruments with which to intervene in the transport system, such as infrastructure extensions, dynamic traffic management, regulations and the like. In this thesis we restrict ourselves to a single instrument, namely road pricing in the form of levying tolls from individual drivers for their actual road use, although it is well known that road pricing measures often are introduced in a package together with other measures (e.g. improvement of public transport such as in congestion pricing in London, electronic tolling in Singapore, or value pricing in San Diego, California). For an overview, see Verhoef et al. (2004).

We assume that the road authority applies some predefined conditions about the type of application of the road pricing instrument, which we will call 'tolling regimes' in the following. The tolling regime defines and fixes a number of elements of the tolling system such as for example the spatial area within which it will be applied, the way of levying the tolls, the use of revenues, the technicalities of vehicle identification and financial transactions, etc. Within these predefined tolling system characteristics ('regimes') however, a lot of freedom still exists in designing the system so as to optimize its effectiveness given the policy objectives. This pertains for example to the precise locations (roads) where to toll, the periods when to toll, the toll levels for different vehicle types at different locations and during different periods, etc. We call these variable characteristics of the tolling regime the design variables of the tolling system. It is the purpose of a toll system design tool such as developed in this thesis to determine the best values of these design variables so as to optimize the authority's objectives. The toll system design model answers questions such as at which sections of the road network to levy tolls at all, and if yes, during which periods a toll will be levied, and how much during each distinguished period of the day.

In Chapter 2 we will describe possible regimes in more detail and specify the type of design variables that are subject of this thesis.

In choosing a tolling regime and optimizing that proposed regime, the road authority is supported in its decisions by a policy analyst who models the system at hand and produces conditional predictions of the likely impacts of the proposed tolling measures. These predictions constitute a basis for the assessment by the authority of the proposed measure's usefulness. The policy analyst works within the tolling regime conditions and other constraints given by the authority. We consider the situation that the policy analyst has the task to determine the optimal tolling system design of a given regime and to deliver the related desired traffic and travel performance indicators. In order to produce these predictions the analyst has at his disposal a toolkit of models that describe the system at hand including models of the physical system (road network with link capacities and link performance functions), network flow models and travel demand models including travel choice models such as for route choice and departure time choice.

In this thesis we represent the position of the policy analyst for which policy objectives, instruments, and tolling regimes are given. We develop a modeling system capable of producing an optimal tolling system design for a road network by determining best values for a number of tolling design dimensions (links, periods, levels, etc.) given an objective function, and producing the related travel demand and traffic flow performance indicators needed in the authority's decision making. The modeling system developed consists of three parts:

1. a flexible mathematical optimization program suitable for all kinds of objectives in which the tolling design variables to be determined are the decision variables;
2. a travel demand prediction model being linked to system ad a. consisting of a set of linked travel choice models describing the likely responses of individual travelers to changing network conditions, in particular to the introduction of (dynamic) tolls;
3. the dynamic network model including the representation of dynamic tolls and a demand-dependent flow propagation mechanism.

1.4 A multi-actor perspective on the road pricing policy problem

The description given above may have clarified that we have a multi-actor view on the road pricing policy problem identifying as main actors: the road authority, the policy analyst, the travelers, and society (see Figure 1.2). These actors are in various roles interacting in reality at various levels in planning, such as at the level of preparing road pricing plans as well as at the level of implementing concrete measures in the network. In practice many more actors might be involved such as for example multiple different authorities and multiple different transport operators. For an overview about different actors in road pricing problem see e.g. Verhoef et al. (2004).

The road authority is the decision maker in practice who tries to solve problems related to travelers and society by formulating objectives to attain and by determining instruments with which to solve the problem. The basic tasks of this decision maker can loosely be summarized as follows:

1. determine the problem to be solved (e.g. less congestion);
2. define the objectives to be attained (e.g. 50% reduction of congestion losses or maximum congestion level);
3. determine the instruments to be employed (e.g. road pricing by regime X);

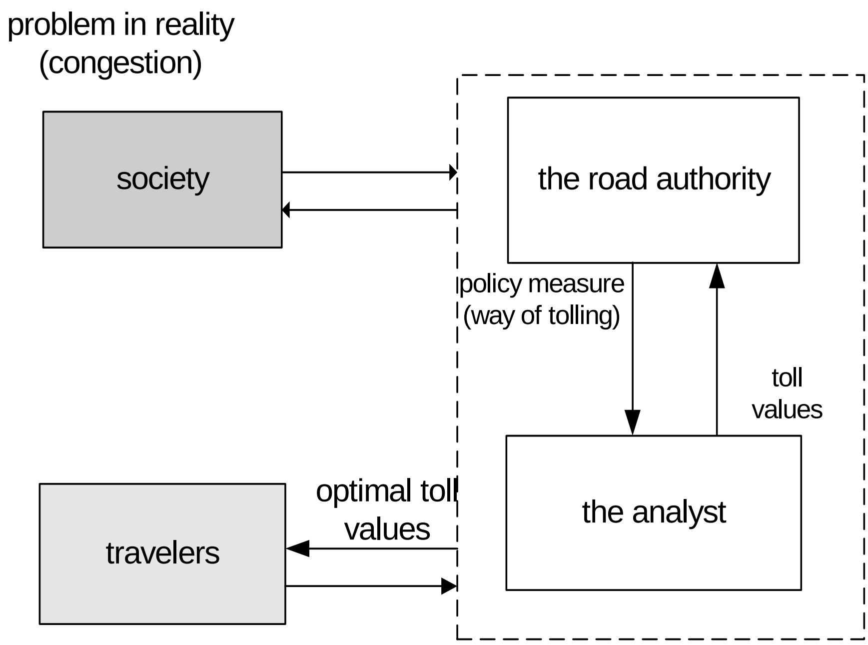

Figure 1.2: Actors in the optimal toll design problem

4. define constraints to be respected by the solutions (e.g. minimum and maximum toll values, exemptions, etc.);

5. define design variables of the chosen regime (e.g. network links, periods, toll levels);

6. determine assessment indicators for the solutions (e.g. level of demand, total network travel time, total congestion delays, emissions, etc.).

These decisions determine the design problem to be solved and form the conditions within which the policy analyst is required to find the best design or set of measures (see Figure 1.3). The decisions of the authority are not made in isolation but will strongly consider the likely responses of travelers and society on these decisions. In that respect these actors maybe considered as players of a game having conflicting interests who try to maximize there own objectives. In Chapters 3 and 4 the interaction process among authorities and travelers will be studied in detail using a game-theoretic modeling approach.

The policy analyst's problem is, given the conditions set by the road authority, to determine the best values of the design variables and to predict the travel demand and traffic flow values resulting from this optimal design. To that end the analyst needs modeling tools for design optimization and travel demand plus traffic operations predictions. Basic elements in the work of the analyst can roughly be summarized as follows:

1. determine a description of the spatial travel demand pattern in the region at hand (a dynamic trip table of origin-destination travel demand over time);

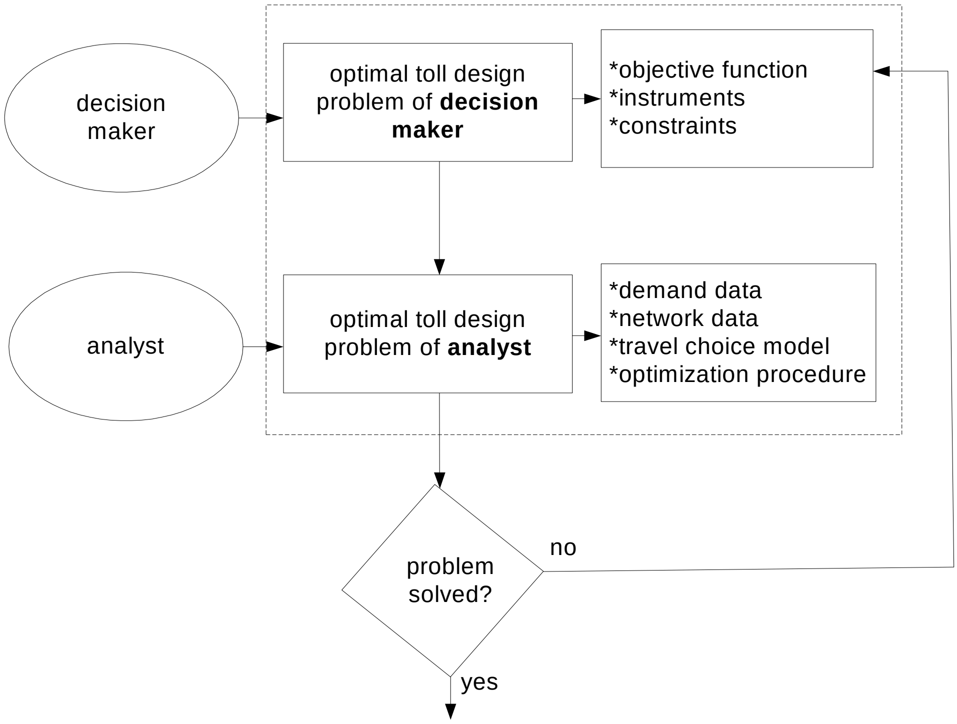

Figure 1.3: The optimal toll design problem (decision maker and analyst aspect)

2. determine a network description suitable for the pricing analysis at hand, allowing toll-dependent and time-dependent propagation of flows over time through the network;

3. develop and apply price-sensitive travel choice models (trip choice, route choice, departure time choice, etc.) that together constitute the overall travel demand model; this demand model is adopted in conjunction with the design model ad 5;

4. specify the values of behavioral parameters to be adopted in the travel choice models;

5. develop and apply a toll system design optimization procedure satisfying the conditions (objective function, constraints, regime type, design variables) set by the road authority. This design tool is a model that tries to adequately reflect the interactions between the authorities decisions on road pricing measures and the traveler responses on such measures if implemented;

6. determine the optimal space-time-level pattern of tolls given the objectives and constraints prescribed by the authority;

7. predict conditional on the toll system design corresponding travel demand and traffic flow patterns and their required indicators for evaluation.

The main contributions of this thesis concern the analyst's activities 3 to 6, with special focus of activity 5. In essence, the analyst tries to adequately model the interactions between the decisions of authorities and travelers on the one hand, and among the travelers themselves on the other hand. In Chapter 2 we will elaborate in more detail on the elements of this activity.

The travelers are a third group of actors relevant in the road pricing policy problem. First of all, in many cases, the car users are the originators of the problems to be solved with road pricing, while at the same time they influence policy makers to take initiatives to remedy these problems. The travelers also are the main group in society whose political acceptance of the proposed road pricing measures is a precondition for their successful introduction.

In addition, the road pricing instrument is meant to be applied to influence the travel behavior of the users of the transport system. Indeed, the travelers will not simply accept the higher trip costs due to the tolls but will somehow try to adapt their behavior in order to minimize the burden induced by the tolls (see e.g. Van Amelsfort et al. (2005a), **?**, Verhoef et al. (2004)). In general, travelers have a wide gamut of options available to achieve this, such as adapting trip making decisions (mode, destination, route departure time choices), location choice decisions (home, work, leisure, etc.), mobility choices (car ownership), and activity choices (work participation, leisure, etc.).

The travelers themselves are assumed to act as selfish individual players with individual preferences and objectives competing for the best services (travel costs) in the network. This implies that the travel decisions of travelers are not independent but are mutually dependent mainly governed by the scarce capacity in the network.

In the context of this thesis we confine ourselves here to the daily trip decisions, in particular whether to make a car trip or not, and route choice and departure time choice if making a car trip. Most of the other types of decisions are subject of other research projects within the MD-PIT program (see Tillema (2007), Ubbels (2006) and Steg et al. (2006)).

The outstanding challenge in predicting the likely impacts of road pricing measures is in adequately modeling the likely travel decision shifts of individual travelers in response to the toll prices and to the travel decisions of the other travelers. In this thesis we adopt such travel choice models that have been developed in a parallel research project in the MD-PIT program (see Van Amelsfort et al. (2005a), Van Amelsfort et al. (2005b)).

Society at large (representing the public, the companies, and other societal forces) is a fourth player in road pricing policy development by influencing the adoption of road pricing by identifying problems (e.g. environmental burdens), by requiring effective solutions, by posing constraints to solutions and to the application of toll revenues. In public policy analyses of road pricing proposals therefore a wide range of societal impacts of such proposals (environmental improvements, welfare gains, etc.) constitute an important element (see e.g. Kalmanje & Kockelman (2004)).

In summary, in our thesis we will first try (in Chapters 3 and 4) to get better insight into the interactions at individual, microscopic level between the design decisions of the authority on the one hand and the travel decisions of the individual travelers on the other hand following from choosing and implementing particular road pricing measures. Having these microscopic insights about the type of problem we then develop as an analyst an optimization model for solving the design problem that adequately reflects the complex interactions among authority and travelers at macroscopic level.

1.5 Research issues of the thesis

The objective of this thesis' research is the development of a modeling methodology capable of determining optimal toll settings (locations, times, levels) given a tolling regime.

The main research issue of the thesis is the development of an optimization model for the design of tolling measures in a dynamic transport network given a tolling regime and other conditions. The considered design variables of the tolling measure (the unknowns of our problem) are the locations, the periods, and the levels of the tolls to be levied, specified for various user classes of travelers. In our view the dynamic property and the multi-user class applicability are strongly required given the dynamic demand and network conditions prevailing in reality. Equally, since travelers strongly differ in their price sensitivity and valuation of travel time losses and schedule delays, a distinction in user classes is deemed highly necessary.

A challenge in this development task is a consistent mathematical formulation of this optimization model that takes, among other matters, the following requirements into account:

- the multidimensional user-class specific responses of travelers to varying locations, periods, and levels of tolls;
- the spatial and temporal dynamics in travel demand;
- the dynamics in flow propagation through the network.

Most outstanding of this research issue is the time-dependent and maybe flow-dependent dynamic nature of the tolls. This dynamic multi-user class approach to tolling is distinctive from almost all studies so far (see Chapter 2).

A second issue to be dealt with concerns the mathematical properties of this optimization problem. The question is whether this optimization problem has a unique solution and whether solution procedures exist that efficiently will find solutions for the optimal design. The thesis will however not address the development of new solution approaches for this problem. For an overview, see Yang & Bell (2001), Clegg & Smith (2001) and Yang & Huang (2005).

As a prerequisite to the development of the optimization model, better insight is required into the system states that may result from the interdependencies among the decisions of the authority (toll settings) and the travelers (travel choices). While it seems impossible to study this in reality (see however Yang & Huang (2005), for a proposal), it seems possible to study this behavior in an experimental setting using a travel simulator (see for example on travel simulators with which responses can be studied experimentally (see e.g. Bogers et al. (2005)). We prefer however a purely theoretical approach to this issue based on microscopic game-theoretic formulations of the problem.

A third important issue is the application of the toll design optimization model to networks. The purposes of these applications are to get insight into the likely impacts of variable tolls on the space-time patterns of the flows and on computational characteristics of this optimization problem. Due to limitations in the availability of efficient solution algorithms our applications will be limited to small hypothetical networks.

The model development in this thesis (and therefore also its applications) will confine the dynamics in the problem (demand, tolls, flows) to the within-day dynamics in a transport network, for example within a peak period. Consideration of the day-to-day dynamics, how important this may be, is a subject left for future research.

1.6 Scientific and practical contributions of the thesis

This thesis contributes to the state of the art in transportation theory in various respects.

We concisely summarize the current state of art pertaining to the theory and modeling of tolling in dynamic networks.

We extend a number of notions with respect to tolling to the situation of dynamic tolls to be applied in dynamic networks with dynamic demand. This refers to possible objectives, road pricing regimes, and road pricing measures.

We formulate the elastic dynamic network equilibrium problem, being a subproblem of the toll system design problem, for dynamic tolls and for multi-user classes with different travel choice behavior especially with respect to price and time sensitivity. This equilibrium formulation applies a simultaneous formulation of the trip, route, and departure time choices.

We formulate a fairly generic toll system design optimization model for use with dynamic tolls in dynamic networks with dynamic demand given a toll regime and other conditions. This optimization model includes a fairly generic dynamic equilibrium model of travel demand.

The plausibility and feasibility of the toll design model are demonstrated with a series of experiments on a number of small networks with varying objectives, toll regimes, and user-class properties, giving clear insights into the impacts of such exogenous factors on the design outcomes. In all these experiments, departure time choice adaptations and flow propagation in the networks are essential new elements.

Another stream of scientific contributions are the in-depth microscopic analyses of toll pricing using game theory. A number of game-theoretic formulations are given of the optimal toll design problem assuming utility maximizing behavior of all individual travelers. These analyses show the differences in outcomes resulting from different assumptions on the interactions among involved actors (authority versus drivers) in the different game types adopted (Monopoly, Cournot, Stackelberg).

Of scientific and practical relevance are the outcomes that clearly show the highly different outcomes resulting from different policy objectives.

From the design model formulation and related experiments it appears feasible to determine optimal toll settings in practice if regimes are given and the travel demand is known.

1.7 Set-up of the thesis

Apart from this introduction and the concluding chapter, the thesis is divided into four parts (see also Figure 1.4)

The first part (Chapter 2) is a problem analysis deepening the specifications of the crucial elements in the toll system design problem and motivating important research choices such as with respect to the dynamic perspective and multi-user class distinctions. Based on descriptions and explanations of possible objectives, toll regimes, roles of actors, etc. a conceptual design of the toll design problem is given as a preparation to a formalized description in following parts of the thesis. This first part includes a concise state-of-the-art.

The second part (Chapters 3 and 4) explores the characteristics of the toll design problem seen from a microscopic perspective looking at individual drivers. It gives an operationalization of the multi-actor view on the toll design problem given in Section 1.4 of this introduction. After introducing the relevant game-theoretic notions, we establish a game-theoretic formulation and specify various game types (Monopoly, Cournot and Stackelberg) to understand more deeply the consequences of different assumptions about the behavior of the authority and that of the drivers. While Chapter 3 gives a conceptual analysis of the toll design problem in game-theoretic terms, Chapter 4 mathematically formulates a number of game types with applications to a small hypothetical network for different objectives of the authority.

The third part (Chapters 5 and 6) deals with the mathematical specification of the toll system design optimization model in macroscopic terms, that is, looking at flows instead of at individuals. In Chapter 5, the design objective and constraints are formalized. The optimization problem type is classified as a so-called MPEC-problem, that is, a mathematical program with equilibrium constraints. These constraints refer to the assumed dynamic equilibrium of the flow pattern in the network for given network properties including tolls. The specification and modeling of this network equilibrium problem is the

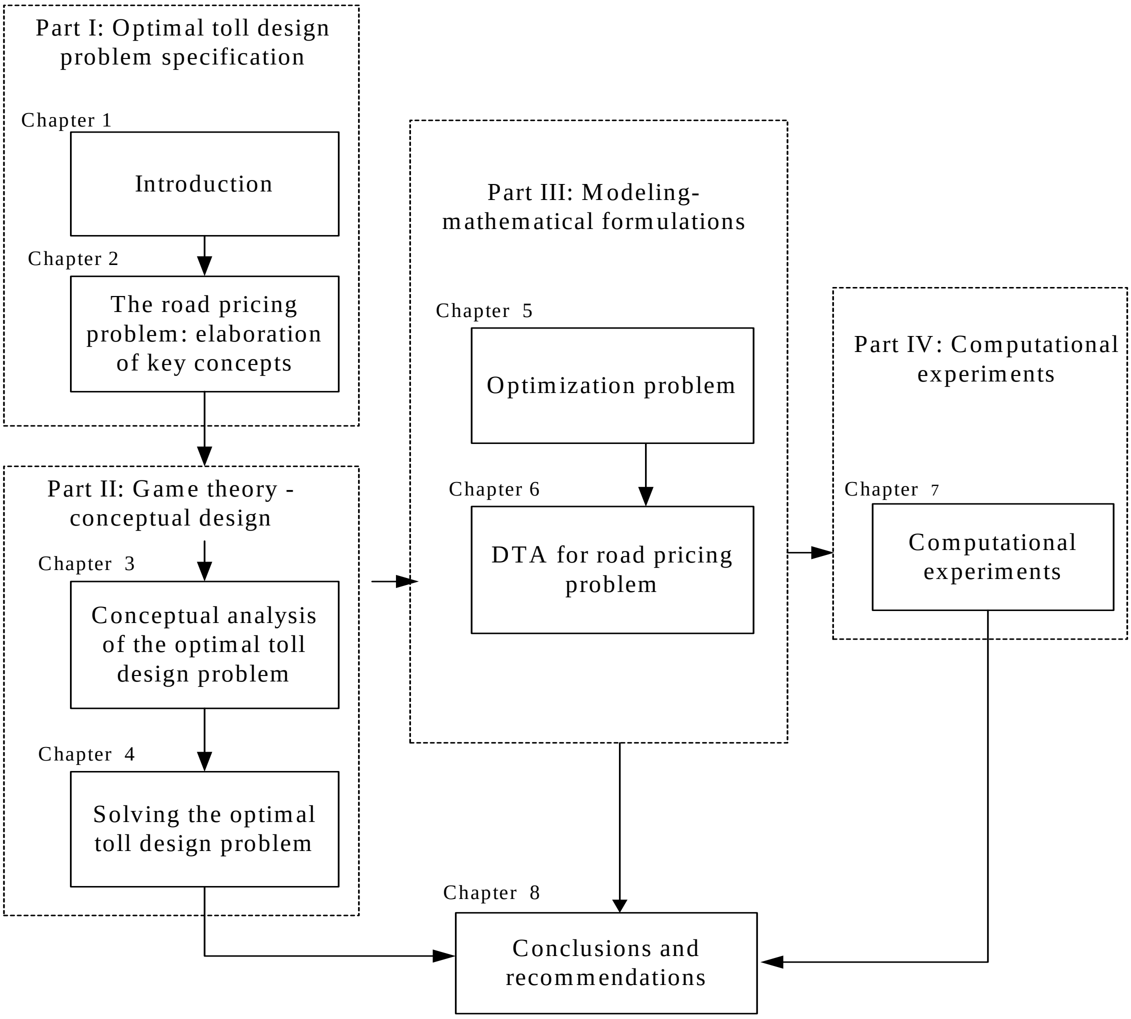

Figure 1.4: Overview of the thesis chapters

subject of Chapter 6. This chapter elaborates in detail the modeling of travel choices (route and departure time choice) in response to tolls and other trip costs.

The fourth part of this thesis (Chapter 7) is about applications with the established models. In order to demonstrate plausibility and feasibility of the developed approach a series of computational experiments are conducted on a range of small hypothetical networks under varying conditions with respect to objectives, assumed choice behaviors, and the like, with special attention to the dynamics in travel demand and flow propagation in response to the tolling.

Chapter 2

The road pricing design problem: elaboration of key concepts

2.1 Introduction

This chapter will elaborate on the tolling design problem introduced in the first chapter, as a preparation towards an operationalization of a mathematical solution methodology in later chapters. The crucial elements of that problem will be clearly defined and analyzed one by one with special attention to the dynamic network context of the problem.

We take the rational policy analysis framework with expressed objectives of the involved actors as a point of departure with special attention for multi-actor context.

We assume in the following the existence of a single road authority responsible for a road network where road pricing is an intended policy instrument to achieve some objective of the authority. This assumption leaves open the case that the authority's plans and decisions in fact follow from some higher-order more or less democratic decision making process with multiple kinds of actors involved.

Since designing an optimal tolling regime is the objective of this thesis, we devote ample attention to the characteristics of the involved design variables.

The chapter discusses already to some extent strategic methodological choices concerning the modeling approach such as concerning demand and supply characteristics in the transport system.

The main contribution of the chapter is the establishment of the proposed approach to the design optimization problem.

The chapter is divided into two parts. Sections 2.2 to 2.4 concern a further elaboration of the policy problem giving more detailed attention to objectives, constraints and design variables of the authority. The remaining sections deal with the research approach to be followed in the thesis in tackling the tolling design problem by specifying in a more formal way the characteristics of the design variables and the way of modeling of the behaviors of authority and travelers.

2.2 Policy objectives/purposes of road pricing

Road pricing enjoys widespread application nowadays all over the world. It should however be noted that the purpose of each of these applications may be highly different. A tentative classification of these different objectives may result into several classes as indicated in Table 2.1. For an overview of toll systems see e.g. Lindsey & Verhoef (2001), Verhoef et al. (2004).

In the case of revenue generation the prime purpose is to collect money from road users, be it for investments in new road infrastructure to be build or for covering the costs of existing tolled infrastructure or just for general purposes. In the latter case, the users of that infrastructure are tolled. Toll levels are set so as to maximize the revenues within a certain span of years, implying a level so as to attract as much paying drivers as possible. This is contrary to most other purposes of road pricing where the objective is to set toll levels such that a sufficiently large proportion of users shift away from the tolled road towards other roads or other travel alternatives (e.g. other modes, other times, no trip at all). Car charging, peak traffic reduction, and congestion charging are tolling applications with the prime purpose of making car use less attractive. Additional underlying aims of car use reduction are better use of existing capacities, environmental improvements, higher traffic safety, revenue generation, and the like.

A completely different way of road pricing is so-called value pricing aimed at attracting car users to the tolled facility. Parallel to a non-tolled congested facility a tolled, guaranteed non-congested facility is offered. Car drivers who are willing to pay the toll will have a much better transport service in terms of guaranteed minimum travel times, no delays, high reliability, etc. than the non-tolled alternative. Tolls vary over time dependent on the congestion conditions on the non-tolled parallel road. Both the toll road operator and the toll road users benefit from value pricing as well as the travelers on the parallel route.

Apart from the adopted objectives mentioned above, road pricing plans might be motivated by lots of other objectives, single or in combinations, such as for example environmental improvement (less emissions or shifting of emissions to another place), safety improvement, higher throughput, etc.

Most important consequence of the chosen objective is the resultant best set-up of the tolling system. It appears that an adequate set-up of the tolling system (regime, locations, periods, levels, etc.) strongly depends on the tolling purpose.

In this thesis we will develop a modeling methodology enabling the determination of the optimal tolling design given an authority's objective and conditions.

We assume that the road authority responsible for a road network has some objective expressed that can be translated into a mathematical objective function. The objective

Table 2.1: An overview of policy objectives

	Objective:	Definition of objective	Explanation	Desired travel behavior	Application in practice
1a	Revenue generation	To collect money to build new (future) infrastructures	Tolling scheme and fare levels are chosen to max. revenues. The aim is to maximize num. of people using road. Fixed tolls per time and space are applied	Shift to other routes and modes are not desired. To attract as much drivers as possible to the tolled roads	North Europe
1b	Revenue generation	To collect money of existing infrastructures for finance of that infrastructure			West and South Europe (Spain, France)
1c	Revenue generation	To collect money for general purposes			
2	Congestion pricing	To reduce traffic congestion to desired levels	The aim is to spread out travel pattern in time and space in order to relieve congestion	Shifts to other time periods, routes and modes are desired	In France (summer periods, weekend days), London
3	Value pricing	To offer special favorable travel conditions to travelers	Drivers have an opportunity to use a dedicated lane parallel to non-tolled congested lane if they are willing to pay. Varying toll	Shifts to another tolled route is desired for the company that provides services	USA (California)
4	Peak traffic reduction	To reduce travel load in peak time periods	To change peak hours. Step-tolling or variable	Shifts to other time periods	Not applied
5	Car charging	To reduce car using at all	To make car using expensive	Shift to other modes	Asia
6	DTM (Dynamic Traffic Manage.)	To improve travel conditions on the network	The aim can be e.g. to improve regulating of intersections,	According to the aim	

might be a single network quantity (e.g. minimize total travel time or total queuing delay, maximize vehicle throughput or maximize revenues) or may be a weighted combination of such quantities. Additionally, the objective may refer to the full network or to parts of the network only (e.g. freeways), or to all users or to a subset only (e.g. person cars).

Since the tolls affect the network flows and their properties (travel times, speeds, user class composition, etc.) the variables in the objective should somehow be derivable from the network flows.

The type of objective chosen might influence whether a clear unambiguous solution to the design task exists.

Throughout this thesis we will adopt as examples a few fairly simple and straightforward objectives (total network travel time minimization, total revenue maximization, and the like) although the design optimization methodology is generic and allows more complex objectives.

The design optimization is conditioned on all kinds of external constraints that the authority may pose on the solution (see Section 2.3). In addition, the authority already has pre-determined the type of tolling regime to be applied which means that some design dimensions are already fixed (for example whether cordon or area tolling, or whether passage-based, distance-based or time-based tolling). An overview of tolling regime types and their dimensions is given in Section 2.4.

2.3 Conditions and constraints

An authority may require from the tolling system to be designed such as to satisfy all kinds of conditions the authority finds important. Some of these conditions may have direct consequences for the design solution (for more information see Brownstone et al. (2003) and Johansson & Mattsson (1995)).

One category of conditions relate to user's acceptance such as:

- comprehensibility of the fare system;
- sufficient availability of travel alternatives (routes, modes, times, destinations);
- transparency of the pricing system.

A further group of conditions may pertain to the investment and maintenance costs of the tolling system.

Another category of conditions may pertain to societal issues such as:

- perceived fairness and equity;
- cost-effectiveness of the tolling system;
- environmental impacts (e.g., due to rerouting)

We assume as self-evident that the tolling system is technically sound causing no traffic disruptions.

While some conditions may require limits to how many toll locations may be established, where tolls can be levied, and with which minimum and maximum toll levels, and toll steps, other conditions may require to limit differences in toll levels to be imposed at different places or to different travelers.

In the design optimization model to be developed, the authority's conditions are translated into mathematical constraints to the optimization problem limiting the values of the design variables.

In our modeling approach we take account of possible constraints that may limit:

- the road sections where tolls might be applied;
- the periods when tolls might be applied;
- minimum and maximum levels of the fares or tolls;
- minimum and maximum step sizes in changing fare values between successive periods;

All this may be specified by user class.

Apart from the constraints on the tolling system design externally imposed by the authority, there are endogenous constraints following from the (assumed) properties of the traffic flows such as the equilibrium conditions. The conditions for societal support of tolling are discussed in Johansson & Mattsson (1995).

2.4 Tolling regimes

We define as tolling regime the way how fares and derived tolls are defined, and how they are levied and collected from the road user during his trip in the network (for an overview see e.g. Gomez-Ibanez & Small (1994), and Lo & Hickman (1997)). The type of tolling regime determines the type of design problem and design variables. The tolling regime has multiple dimensions to consider as will be summarized below.

First-of-all the fare base. The fare to be paid by the road user maybe based on

- passage,
- distance traveled,
- time spent.

The usual road toll systems are passage-based: a fixed fare is levied for each passage, possibly classified by vehicle type and time-of-day. On most toll roads the fare to be paid is distance-based. For distance-based and time-based fares, the trip toll is determined by the product of the unit-fare and distance traveled or time spent on the tolled infrastructure.

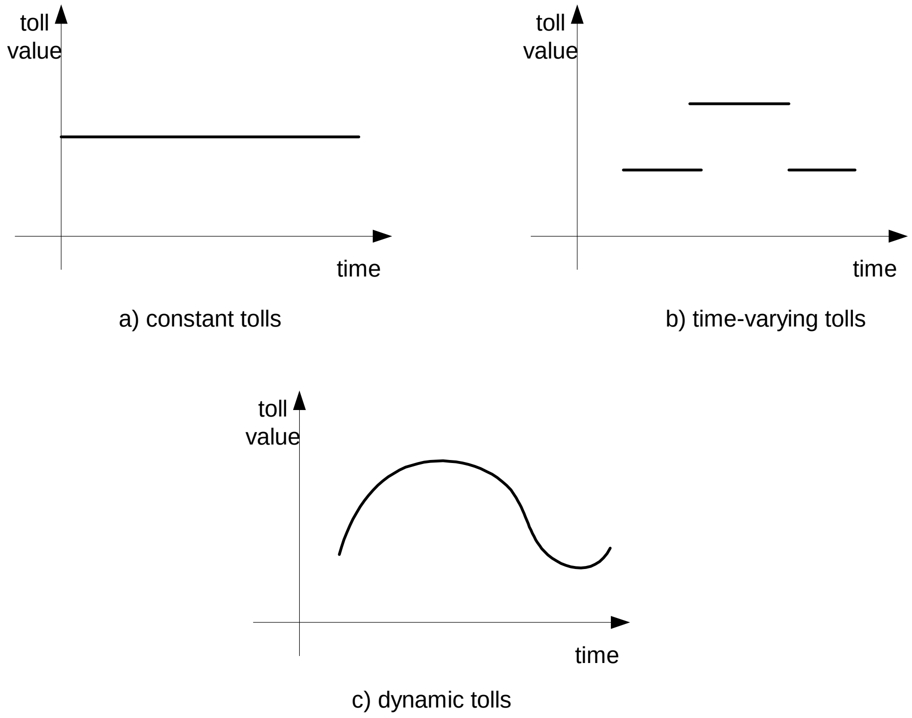

Figure 2.1: Road pricing temporal analysis

The passage fare or unit-fare maybe a fixed value constant over time or time-varying step tolls where the fare level depends on the time period but is constant within a period. Dynamic fares may vary more or less continuously over time, for example depending on the actual congestion such as in the case of the Californian value-pricing projects (see Figure 2.1).

Apart from time and space, the fares may be dependent on user type such as vehicle type, frequent user, foreigner, and the like.

Another dimension of the tolling regime is the levy base. The trip toll may be based on:

- tolled links used during the trip;
- tolled routes used;
- tolled OD pairs;
- tolled zones that are visited during the trip (such as cordon pricing).

Each levy base can be combined with the fare bases given above to arrive at various types of tolling regime.

We refrain from describing other dimensions of tolling regimes such as revenue use and the technicalities of vehicle identification, information provision, payment, enforcement, and the like, since we assume that the influence of these factors on the travel choice behavior is negligible.

Model-based optimization of the type of tolling regime is outside the scope of this thesis. We assume that the tolling regime is given and that a model-based optimization of the given regime concerns the locations, periods and levels of the tolls to be levied, specified by user-class. Finding the best type of regime may be done using a scenario approach.

The opinions in society as well as the technological opportunities more and more favor distance-based fares for reasons of effectiveness, fairness, etc. Although our modeling system will facilitate various kinds of tolling regimes, we adopt in this thesis in our examples distance-based fares, having the advantage that these may be easily be translated into link-based passage-fares. More about different pricing measures can be found in (DePalma & Lindsey (2004*a*)).

2.5 Operationalization of possible tolling regimes

In this section we describe some possible different tolling regimes a network user may experience while traveling. First, the explanation and illustration of different tolling regimes are given, after which the mathematical formulations of different tolling regimes are presented.

2.5.1 An overview of some possible tolling regimes

As stated before, the optimal toll design problems can be considered from at least four different dimensions: time, space, fare level and user-class. Moreover, different classifications can be made with regard to different levels of variation for each of these dimensions. Therefore, taking into account the time and space perspective of the optimal toll design problem, there are several tolling regimes which can be applied on the network. For clarification, we will present a sample of them below:

1. (a) **A fixed fare charged at entrance on the network** (space perspective)

   With this tolling regime a fixed fare is charged to a user when he enters the tolled network segment irrespective of travelled distance or time spent. Note that the fare is constant and also not dependent on time of the entrance to the network (see Figure 2.2, case 1*a*).

   (b) **A fixed fare charged to the user at exit of the network** (space perspective)

   Similar to the previous case, with this tolling regime a user is charged when he/she exits the tolled area. The charged toll is constant over time periods (see Figure 2.2, case 1*b*) and is independent of travelled distance or time spent.

2. **A variable fare at entrance depending on time of entrance** (time perspective)

   (a) With this tolling regime the travelers will experience different fares depending on their actual entry time to the network. Clearly, in contrast to previous case,

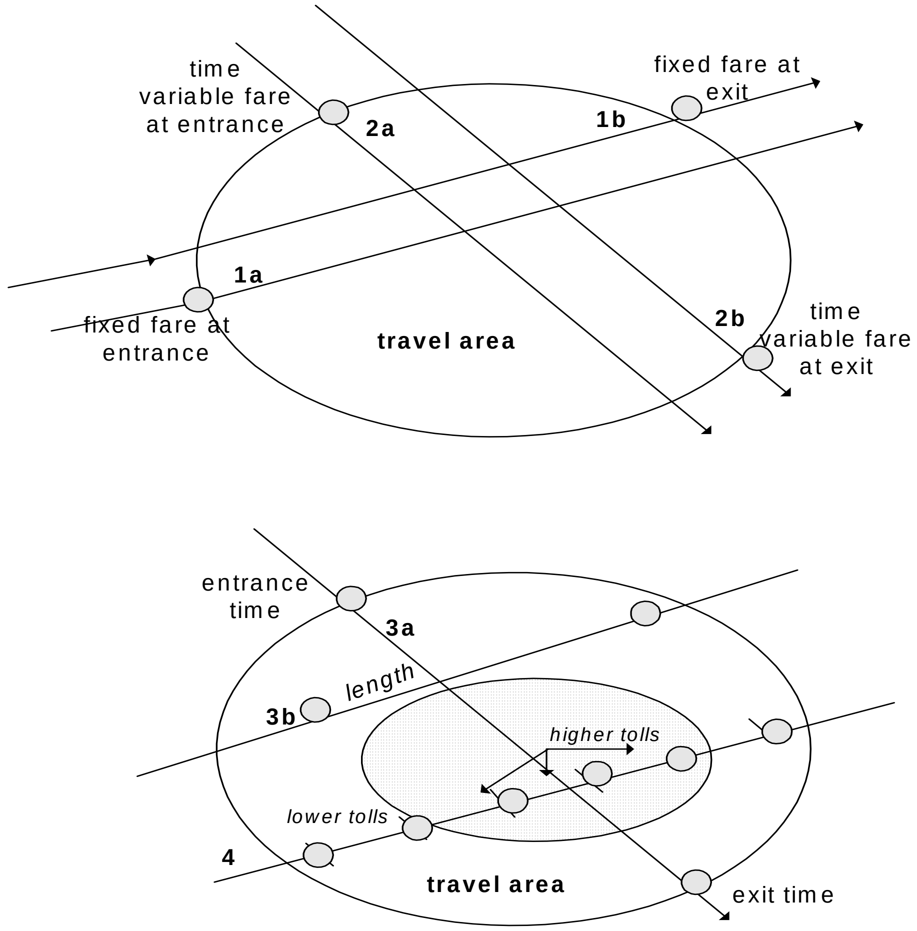

Figure 2.2: An illustration of different tolling regimes

Table 2.2: Tolling regimes

	time dimension			
space dimension		fixed fare	variable fare	flow dependent fare
	exit/entrance area	$1a, 1b$	$2a, 2b$	
	trip length en duration in area		$3a, 3b$	
	single links		4	

travelers are motivated to change their entry time in order to avoid to pay tolls (see Figure 2.2, case 2*a*).

(b) **A variable fare at exit depending on time of exit of the traveler** (time perspective). With this tolling regime a different fare will be charged dependent on time of exit of the network (see Figure 2.2, case 2*b*).

3. **Fare dependent on the length or duration of the trip** (space or time perspective)

   The toll is charged depending on the length or duration of the trip in the tolled network (e.g. kilometers driven) irrespective of 'where' in the network and 'when' (See Figure 2.2, cases 3*a* and 3*b*, respectively). Usually, the toll increases with the trip length and duration. It should be noted that if the toll is dependent on the duration of the trip, the entry and exit times are relevant.

4. **A space variable fare depending on the periods of traveling** (time, space and duration dependent)

   Suppose that in the different time periods different fares are imposed (usually higher in the peak period and lower on the shoulders of the peak-period). Thus, on different links on the travel area different fares are set (see Figure 2.2, case 4). With this tolling regime, a user is charged depending on his time of traveling and fares imposed in these time periods and in travel area. In other words, the total trip toll is the sum of fares over all links and time periods. This tolling regime is the most complex in this classification.

It should be noted that all tolling regimes can be applied differently for *different user classes* but also for different road types.

A graphical interpretation of previously stated tolling regimes is shown in Figure 2.2. We consider a specific path p through the travel area given in Figure 2.2. The total trip toll is the sum of fares charged over different parts of the route. The fixed fare is illustrated using cases 1*a* and 1*b*. The variable fare is illustrated using cases 2*a* and 2*b*. The toll dependent on the length of the trip is given by case 3*a*, while the toll dependent on duration is given with case 3*b*. The combination of time and space is given in case 4 (where, e.g. two different tolls should be paid).

A classification of tolling regimes per time and space is given in Table 2.2.

In Table 2.2, the simplest cases regard to the space and time distinction are presented first (exit/entrance area). The most complex case is the space variable fare: the trip fare depends on the places of the user. For example, in a city center, the fare can be different from that outside of the center. Therefore, fixed, variable and flow dependent fares are considered. Namely, every single block in Table 2.2 determines a different tolling regime for a specific space category.

Some examples in practice can be noted: congestion pricing in London (TfL (2004)) as an example of exit/entrance tolling with fixed fare (Hensher (2003)); dynamic pricing in California (Brownstone et al. (2003)) is an example of exit/entrance tolling regime where the fares are flow dependent; tolling of trucks in Germany is an example of trip length tolling regime. Electronic road pricing schemes in Singapore and Hong Kong are examples of time and location dependent fares.

A mathematical formalization of the described tolling regimes is given in the following subsection.

2.5.2 Mathematical formulations of different tolling regimes

Let us introduce the following notation: t - time period on a time scale (a time period can be tolled or not-tolled), k- departure time period of a traveler; t_m^{ent}-time period in which traveler m enters the network; t_m^{exit} the time interval in which the traveler m exits the network, where the following equation holds: $t_m^{exit} = t_m^{ent} + \tau_m$, where τ_m denotes travel time $(\tau_m(t_m^{ent}))$. Let us define T as the studied time period (that include all tolled and non-tolled periods).

Let θ_{trip} denote the total trip toll charged per trip, while $\theta\,(s,t)$ indicates the fare charged at location s during time period t. The fare θ can be:

- constant fare in all time periods, $\theta\,(s,t) = \theta\ , \forall t \in T$;
- variable fare $\theta\,(t)$, different in different time periods t. In our experiments (see Chapter 7), this fare pattern results from the computation of a single maximum fare value $\overline{\theta}$ according to the given proportions $\phi(t)$, that is $\theta\,(t) = \phi(t) \cdot \overline{\theta}$.

Taking into account the previous classification, the following mathematical formulations arise:

1. (a) **A fixed fare charged at entrance on the network** (space perspective)
   With this tolling regime, the trip toll is the function of the fare only.

   (b) **A fixed fare charged to the user at exit of the network (space perspective)**

   Similar to previous, the total toll is a function of fare only. Therefore, the mathematical expression for both tolling regimes ($1a$ and $1b$) is:

$$\theta_{trip} = F\,(\theta) = \theta \tag{2.1}$$

(a) A **variable fare at entrance depending on time of entrance** (time perspective)

In this case, the trip toll is a function of the fare valid at the departure time of the traveler. With this tolling regime, the time of entry of the traveler, t_m^{ent} determines the toll to be paid.

$$\theta_{trip} = F(k, \theta) = \theta(t_m^{ent}) \tag{2.2}$$

(b) **A variable fare at exit depending on time of exit** (time perspective)

With this tolling regime the time of exit of the network t_m^{exit} determines the toll to be paid:

$$\theta_{trip} = F(k, \theta) = \theta\left(t_m^{exit}\right) \tag{2.3}$$

2. **Toll dependent on length of trip only or part of trip** (space perspective)

   If L denotes the length of the trip, then

$$\theta_{trip} = F(L, \theta_l) \tag{2.4}$$

   If the fare is constant per length unit of the journey, then

$$\theta_{trip} = L \cdot \theta_l \tag{2.5}$$

3. **Toll dependent on duration of the trip or part of trip** (time perspective)

   If D denotes the duration of the trip, then

$$\theta_{trip} = F(D,\theta_d) \tag{2.6}$$

   If the fare is constant per time unit, then

$$\theta_{trip} = D \cdot \theta_d \tag{2.7}$$

4. **A time variable fare depending on the periods of traveling** (time and toll perspective)

   Let us with I_t denote if traveler m travels in the time period t, thus the value of I_t is 1, otherwise 0. Note that $I_t = f\left(t_m^{ent}, \tau\right)$. While $\phi(t)$ denote the given proportions of fare distribution per time periods, $\overline{\theta}$ is the single resulting maximum fare value to be computed, then the total trip toll can be expressed in the following way:

$$\Theta_{trip} = \sum_{t=1}^{n} I_t \cdot \phi(t) \cdot \overline{\theta} \tag{2.8}$$

   where n is the total number of time intervals.

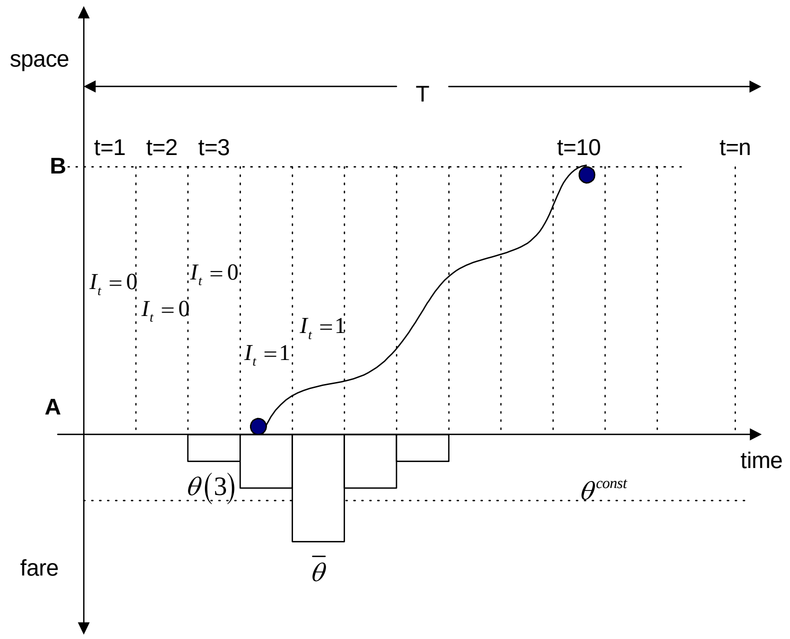

Figure 2.3: An illustration of possible tolling regimes with constant and variable fares

It should be noted that for some time periods the fares may be zero (dependent on ϕ proportions). However, the total trip toll is the summation over all time periods of the trip. We illustrate the previous formulation with Figure 2.3.

In Figure 2.3 the time axis represents the different time periods, $T = n \cdot t$, where n is the total number of time periods. On y axis the space is denoted (in our case route from place A to B). Let us denote the fare imposed in time interval t on path between A and B as $\theta(t)$. Note that on the fare axis, two different tolling regimes are illustrated (constant tolls θ^{const} and time-dependent tolls $\theta\,(t)$, where $\theta\,(t) = \phi(t) \cdot \overline{\theta}$). With a black solid line the trajectory of traveler m is illustrated. Note that for traveler m the following values hold: t_m^{ent}, t_m^{exit}, and k_m [?]. Thus, the time period in which the traveler m enters the network is $t_m^{ent} = 4$, the time period in which the traveler exits the network is $t_2^{exit} = 10$, while $I_k = \{0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0\}$ are the values of I_k indicating time intervals in which the traveler m traveled (value 1) or not (value 0). Thus, the number of traveled time intervals from A to B is 7. Note that time interval 4, 5, 6, and 7 are tolled time intervals while in time intervals 8, 9 and 10 fare is zero. Using expression (2.8) the trip toll can be computed using given values $\phi(t)$ and computed value for $\overline{\theta}$ per time period.

2.6 Problem types for road pricing studies

In this section different problem types for road pricing studies are described:

1. the type of tolling regimes considered in the study;
2. the type of OD-demand considered in the analysis;
3. the type of traffic flow analysis considered in modeling.

While the first issue is a problem of the authority, the other two issues are problems of the analyst how to design his policy study.

With respect to the **tolling regime** (1) we will distinguish the following cases:

1. (a) constant tolls over time of the day and also from day-to-day;
   (b) time-varying tolls over the day, the time and money pattern of which is constant from day-to-day;
   (c) fixed time-varying tolls over the day, the pattern of which may change from day to day;
   (d) flow-dependent varying tolls over the day that therefore will change from day-to-day (control problem).

   Case 1*a* is considered a reference case.

   Case 1*b* is considered useful for comparative purposes.

   Case 1*c* is considered relevant if one requires that the toll levels are available to the public early enough (one day earlier). These levels may be calculated off-line during the night on the basis of a demand prediction based on known historic demand. This presupposes an estimation of a dynamic OD-table for the next day.

   Case 1*d* describes the situation where toll levels depend on prevailing demand that is continuously monitored or estimated from observations.

With respect to the **OD-demand** adopted in predicting network flows and optimizing tolls (2) we distinguish the following cases:

2. (a) a static OD-table assumed constant over time considering only route choice adaptations;
   (b) a fixed (exogenously given) dynamic OD-table, however being constant over the days; daily spatial flows are assumed constant;
   (c) a variable (endogenously determined) dynamic OD-table that may change from day-to-day; however, daily spatial flows are assumed constant (route and departure time choice adaptations);
   (d) a space-time variable dynamic OD-table that may change from day-to-day (because of adaptations in trip, mode, departure time, and destination choices).

Table 2.3: Specification of the problem type of optimal toll design problem

network flow assignment		tolling regime		
		static	dynamic	
			time varying	flow dependent
static OD demand	static	Hearn(2002) Yang (1999)		
fixed OD demand	static	Hearn(2005)		
	dyn		Joksimovic(Ch 7)	Viti et al.(2003)
variable OD demand	static			
	dyn		Abou-Zeid(2003) Joksimovic(Ch 7)	Mahmassani(2005)
elastic OD demand	static	Joksimovic (Ch 4)		
	dyn			

Finally, we may distinguish different ways of **modeling network flows** (3): First of all modeling travel choices in networks can be done in a deterministic or probabilistic way of which the latter is a far better representation of reality, especially if road pricing is involved because of known heterogeneity in the population with respect to time and cost preferences. In addition, the propagation of flows in the network may be modelled as static or more realistic as dynamic and flow-dependent. We thus may distinguish the following network cases:

3. (a) static network assignment with only freedom of route choice;
   (b) dynamic network assignment with only freedom of route choice;
   (c) dynamic network assignment with both departure time and route choice adaptations;
   (d) Elastic dynamic network assignment in which apart from route and departure time choice also the level of demand may adapt, e.g. because of modal choice and/or destination/trip choice shifts.

In all these cases a deterministic equilibrium or a probabilistic equilibrium might be chosen as demand modeling principle.

Combining these three problem dimensions gives rise to the following classification of problem types with respect to dynamic tolling on roads (Table 2.3).

Table 2.3 indicates that most known studies of road pricing are confined to static networks and to deterministic equilibrium.

In the table we have inserted with which types of problem we are concerned in this thesis, namely:

1. elastic static OD-demand with deterministic static equilibrium (see Chapter 4);

2. exogenous dynamic OD demand with dynamic network with probabilistic travel choice modeling and dynamic equilibrium (see Chapter 7);

3. endogenous dynamic OD demand dynamic network with probabilistic travel choice modeling and dynamic equilibrium (see Chapter 7).

So, in all our examples in this thesis (except case 1.) we will apply within-day dynamics of travel demand and network supply, and probabilistic travel choice modeling.

2.7 Summary of literature on road pricing

In recent years, the subject of road pricing gained interest from both economist and transportation researchers. From the practical point of view, the crucial question is how to implement the pricing concept in reality, both in developing technology for efficient charging and gaining political acceptance and positive public opinion (see more in Button & Verhoef (1998)). The problem of road pricing has been studied in the literature from different modeling perspectives and under various assumptions. The (economic) theory of road pricing dates back to Pigou (1920) who used the example of a congested road to discuss optimal congestion charges and externalities. The interpretation of social cost and toll charge dates from Knight (1924). Works on road pricing that deserve to be mentioned here include Wardrop (1952), Walters (1961), Beckmann (1965), Vickrey (1969).

2.7.1 The first-best pricing problems

The theoretical background of road pricing has been related with the fundamental economic principle of marginal-cost pricing. Road users, using congested roads, should pay a toll equal to the difference between the marginal-social cost and the marginal-private cost in order to maximize the social surplus. In other words, the toll that should be paid includes the time losses of other road users, emissions, noise and the like. The social surplus can be seen as a difference between the total costs and the total benefits. For more details the reader is referred to Verhoef (1996). First-best congestion tolls are derived in static deterministic models in the case for homogeneous users and established in general traffic networks. The toll which is equal to the difference between the marginal social cost and marginal private cost is charged on each link on the traffic network. The result is a system optimal flow pattern in the network (Beckmann (1965), Dafermos & Sparrow (1971)). Further, first-best pricing is applied to a general congested network with multiple vehicle types, with link flow interactions, and queuing (Yang & Huang (1995)). Static stochastic equilibrium models in a congested network have been developed by Yang (1999).

Mahmassani & Herman (1984) developed dynamic marginal (first-best) cost pricing models for general transportation networks. As indicated by these authors, the application of their models might not be easy to implement in practice. Moreover, since tolls are based on marginal cost pricing, it is implicitly assumed that all links can be priced dynamically, which is practically infeasible.

2.7.2 The second-best pricing problems

The first-best or marginal-cost pricing scheme seems to be not applicable in practice. The reason is that it is difficult to charge users on each link on the network with regard to the operating cost and public acceptance of such proposal. In the recent literature, so-called second-best pricing appears to be the most relevant approach from the practical perspective of road pricing (Lindsey & Verhoef (2001)). The simplest examples of the second-best pricing problem concerns a two route network, where an untolled route is available (Verhoef et al. (1996), Liu & McDonald (1999)).

A nice overview of different types of toll charging schemes is given in the work of May & Milne (2000) where travel-distance based charging, travel time or travel-delay based charging, link-based charging and cordon-based charging are described. These road pricing schemes are of practical interest. For example, in a link-based pricing scheme, tolls are charged only on a subset of networks links. In these situations, tools are charged at a bottleneck (usually urban bridges, tunnels or expressways). For more information, see e.g. Yang & Lam (1996). To reduce traffic demand in congested urban central areas a cordon-based pricing is introduced (May et al. (2002)).

In the literature, the problem of simultaneous determination of toll locations as well at toll levels is discussed in Hearn & Ramana (1998). Verhoef (2002) examined the selection of individual toll links, and the determination of toll levels using some sensitivity indicators. Yang & Zhang (2002) considered selection of optimal toll levels and optimal toll locations for achieving maximum social welfare using a bi-level programming approach with both discrete and continuous variables. Shepherd & Sumalee (2004) explored the usefulness of solving the optimal toll problem for a medium scale network. In Sumalee (2007), a methodology for defining optimal locations of a multi-concentric charging cordon scheme (for the case of singles user class) is given. The results suggest substantial improvements of the benefit from the optimized charging cordon schemes in a static case.

2.7.3 Dynamic road pricing problem

By far, most of the literature concerns road pricing for static networks for which constant supply and demand properties over time are assumed (for a recent overview see Yang & Huang (2005)). We are convinced that such an approach does not adequately address the dynamic patterns that are characteristic for most networks where pricing maybe a relevant policy option. In addition, most of this ‘static’ literature deals with marginal cost pricing which may be theoretically interesting but fails to represent the type of objectives that road authorities want to achieve in practice. Below we give a concise account of the scarce literature dealing with the road pricing system design issue for dynamic networks (ranging from a single highway bottleneck to a general transportation network).

Bottleneck models

Starting from a standard bottleneck model developed by Vickrey (1969), many authors have made significant contributions by including heterogenous commuters, route choice, elastic demand and time-varying tolls (Braid (1989), Arnott et al. (1993)). In the standard bottleneck model, congestion is assumed to arise when vehicles queue due to the bottleneck. Because of the capacity of the bottleneck, not all travelers will arrive at their preferred arrival times. Their costs are associated with their early or late arrival, which together with toll are added to the cost of the trip. By choosing their departure time the travelers aim to minimize their generalized trip costs. Arnott et al. (1988) discusses schedule delay and departure time decisions of heterogeneous commuters. In Arnott et al. (1990) the effectiveness of various pricing policies (time-varying, uniform, and step-tolling) are compared. Arnott et al. (1992), Arnott et al. (1993), and Arnott et al. (1998) have further extended the bottleneck model.

Dynamic congestion pricing models, where network conditions and link tolls are time-varying have been addressed in Henderson (1974). The approach involves the distribution of travel delays and scheduling of costs of the peak, and the duration of the peak. This type of generic model was further extended by Huang & Yang (1996). Dynamic congestion pricing models with time-varying link tolls by applying a speed-flow function for a congested road are presented in Agniev (1977), Carey & Srinivasan (1993). Mahmassani & Herman (1984), Arnott et al. (1993), Ben-Akiva et al. (1986), Braid (1989) and Mun (1994) conducted equilibrium analysis on bottlenecks with elastic demand. Laih (1994) examined queuing at a bottleneck with single and multi-step tolls. Henderson (1974) incorporated two groups of commuters differing in value of time and schedule delay penalty into a dynamic model. Also Arnott et al. (1998), Arnott et al. (1992) studied various aspects of bottleneck problems with general groups of heterogeneous commuters. Yang & Huang (1997) and Huang & Yang (1996) applied optimal control theory to develop a general time-varying pricing model for a state-varying exit capacity bottleneck (hyper-congestion) with elastic demand. For further developments on the bottleneck models we may refer to Van der Zijpp & Koolstra (2002), DePalma & Lindsey (2002), Zhang, Yang, Huang & Zhang (2005). Hyper-congestion and optimal road pricing in a continuous-time is studied in the work of Verhoef (2003). The bottleneck models with an un-tolled alternative (substitute) were discussed in Verhoef et al. (1996), Liu & McDonald (1999).

The limitations of previous models are that they consider either a bottleneck or a single destination network.

Network models

Several studies have been developed in the literature to deal with traffic congestion and pricing in general dynamic networks.

Dynamic pricing models in which network conditions and link tolls are time-varying, have been addressed in Wie & Tobin (1998), comparing the effectiveness of various pricing policies (time-varying, uniform, and step-tolls). As indicated by the authors, the application of their model is limited to destination specific (rather than link or route based) tolling regimes, which might not be easy to implement in practice. Moreover, since the tolls are based on marginal cost pricing, it is implicitly assumed that all links can be priced. In Acha-Daza & Mahmassani (1996) marginal toll pricing is performed showing that use of variable tolls reduces the total travel times for congested networks.

In the work of Viti et al. (2003) a dynamic congestion-pricing model is formulated as a bi-level programming problem, in which the prices are allowed to affect the (sequentially) modelled route and departure time choice of travelers. Abou-Zeid (2003) developed some models for pricing in dynamic traffic networks where the analytical properties of dynamic pricing models are studied and a sensitivity based method is used to solve the pricing problem in a dynamic framework. Mahmassani et al. (2005) solve the optimal toll problem with heterogeneous users using a simulation approach. In this thesis, the time-varying pricing model including route and departure choice is solved using a simple algorithm for the road authority's objective of optimizing total travel time in the network. Moreover, these models are extended to include different and more general tolling schemes, different objectives of the road authority taking account heterogeneous behavior of travelers.

Heterogenous users

The concept of value of time (VOT) and schedule delays plays an important role in road pricing analysis. In the studies where heterogenous users are present, various network equilibrium models have been developed by assuming a discrete VOT or by continuously distributed value of time across the whole population. Recently, Mayet & Hansen (2000) investigated second best-congestion pricing with continuously distributed VOT for a highway (with an unpriced substitute). The optimal tolls are determined depending on whether the social welfare function is measured in money or in time, and whether toll revenue is or is not included as part of the benefit. Yang & Huang (2004) investigated the relationship between the multi-class, multi-criteria traffic network equilibrium and system optimum problems. Another example of a study that is of interest is study of Verhoef & Small (2004), who consider a differentiation of tolls across parallel traffic lanes by using a static model. Using a product differentiation approach, Verhoef & Small (2004) examined the impact of user heterogeneity (with a continuously distributed VOT) on the results of the first, second-best and the revenue-maximizing pricing policies. DePalma & Lindsey (2004*b*) examined the effect of congestion pricing with heterogeneous travelers using general-equilibrium welfare analysis.

In subsequent chapters we include literature reviews pertaining to the subjects dealt with in these chapters such as on game theoretic modeling of transport flows (Chapter 3), mathematical approach to road pricing (Chapter 5) and dynamic traffic assignment (Chapter 6).

2.8 Research approach in this thesis

This thesis deals with the dynamic multi-user optimal toll design (DOTD) problem concerning the determination of toll values in time and space on a dynamic transportation network that will optimize a given objective of the road authority. We assume the existence of a single road authority responsible for a road network where road pricing is an intended policy instrument to achieve some objective of the authority. The DOTD problem thus is a design problem in which locations, time periods and fare levels are the design variables to be determined. It is the aim of the thesis to develop a modeling methodology for this design problem for which the dynamics in travel demand, network supply, and toll levels are the outstanding features together with the multi-user-class distinction in the demand.

The methodology is directed at the use for strategic planning purposes instead of for operational control. Hence it is assumed that the day-to-day dynamics can be neglected so that the dynamics only refer to within-day variability.

We confine our DOTD methodology to the two main actors in the road pricing system design policy problem, namely the authority on the one hand and the travelers on the other hand. We may describe the interaction between these two actors by a bi-level model.

At the upper level the decisions of the authority are modeled being in our methodology the determination of the best values for the design variables at hand given a tolling regime, possibly determined in an external scenario exercise. These design decisions are governed by the objectives of the authority and his constraints and other conditions. On the other hand these decisions take into account the travel choice adaptations of the travelers that may be expected as a response to the decisions of the road authority with respect to the tolling design variables. This upper level design optimization problem will be tackled by specifying a mathematical program with constraints where the constraints partly come from the authority and partly from the equilibrium conditions of the network flow pattern.

At the lower level the traveler choices are modeled given the tolling system values chosen by the authority. We assume each traveler follows its own objective of subjective maximizing his trip utility in making his travel choices in response to the tolling situation in the network. In principle all travelers are assumed unique in their preferences for time, cost, and other trip attributes which in addition to the consideration of different user classes is modeled by adopting probabilistic choice models for route and departure time choice. The lower level problem will be solved in our methodology using a multi-class dynamic equilibrium approach with a simultaneous route-departure time choice.

All actors at both levels are assumed to behave rationally, that is they try with their individual actions to optimize personal objectives within the constraints limiting their choice options.

We will attempt to develop a mathematical programming approach as the basis for our tolling design methodology.

It is part of this proposed approach to study the solution properties of this design problem, whether there exist single or multiple solutions and the like. For the time being, we will study fairly simple and straightforward objective functions applied by the authority expressed in network variables such as total travel time, total congestion delay, total revenues, and the like. It is subject of future research to study more complex objective functions of the authority.

The proposed design methodology assumes a given tolling regime (see Sections 2.4 and 2.5). Within this regime the following design variables need to be determined:

1. Toll locations. We assume a subset of links given, determined by the authorities constraints, where tolls can be levied (maybe valued equal to zero). This design variable is of discrete nature.

2. Toll periods. In principle this design variable is of a continuous nature. We assume however a given subdivision of the time dimension into small equal-size periods within which the fare level is constant. This is most probably required by practical considerations of implementation in reality as maybe also by computational reasons. See the Californian projects, Brownstone et al. (2003). The fare in periods maybe zero.

3. Fare level. We assume always positive fares. Depending on the chosen tolling regime the fare refers to a passage, to a single link, to unit length or to unit time (see Section 2.4). In principle this design variable is of a continuous nature including the value zero although we may assume that in reality a division in a set of discrete fare classes makes sense. There may be a constraint on the fare difference between successive time periods on each tolled location.

4. User class fares. Fare levels may vary between given user classes to the extent that some user classes may even be exempted from paying toll at all.

The design optimization model uses a travel demand model that describes the aggregate responses of travelers in the network. The travelers are assumed to base their travel decisions (trip, route, time) on individual subjective utility maximization governed by a composite cost function in which all relevant costs to which the traveler is sensitive are included, such as time costs, tolls, schedule delay costs, etc.

We assume that traveler's trip decisions are dependent on the total trip toll which is a summation of the various toll components to be paid during the trip at entries or exits, at links or in zones depending on the fare and levy base (see Section 2.4). We assume a trip time and cost information level of the travelers such that no en-route choice adaptations are necessary so that all decisions are assumed to be pre-trip decisions. This is in line with the strategic planning context chosen for our model instead of an operational traffic control context.

Travelers are individuals that show different preferences and valuations of trip attributes such as speeds, times, delays, and costs. This heterogeneity in the population of travelers is of paramount importance to an adequate prediction and assessment of the response to policy interventions such as road pricing. In addition, policy makers often chose their measures so as to influence specifically certain user classes and require knowing the impacts of their measure specifically for different user classes. In order to arrive at valid outcomes of our research a distinction in user classes that reflects this heterogeneity in policy interest and travel behavior is inescapable.

In our research we distinguish therefore the following dimensions of user classes:

- traffic characteristics: cars and trucks exhibit different speeds, travel time functions, capacity requirements, and the like.
- travel choice behavior: time and costs sensitivity parameters may differ for user groups and trip purpose types.
- toll levels: these may differ for user classes even to the extent that some user classes maybe exempted from paying any toll.

The model we develop is fairly generic and flexible with respect to the handling of user classes. In our examples however, we keep the user-class distinction simple.

The characteristics of the optimal toll design problem are illustrated in Figure 2.4.

In this work only departure time and route choice are taken into account (gray blocks in Figure 2.4).

2.9 Summary and Conclusions

This chapter offered an elaboration of the crucial elements of the road pricing design problem aimed at preparing for the development of a design optimization methodology in the next chapters. Special attention is given to a description of various possible tolling regimes in a dynamic network context. The type of tolling regime determines the type of design variables in the design optimization task. The crucial design variables (the unknowns) in our methodology are the locations where, the periods when, and the levels of the fares to be levied, specified by user-class.

We presented the design problem as a multi-actor decision problem with the authority on the one hand and the travelers on the other hand. Both actors are assumed to act rationally

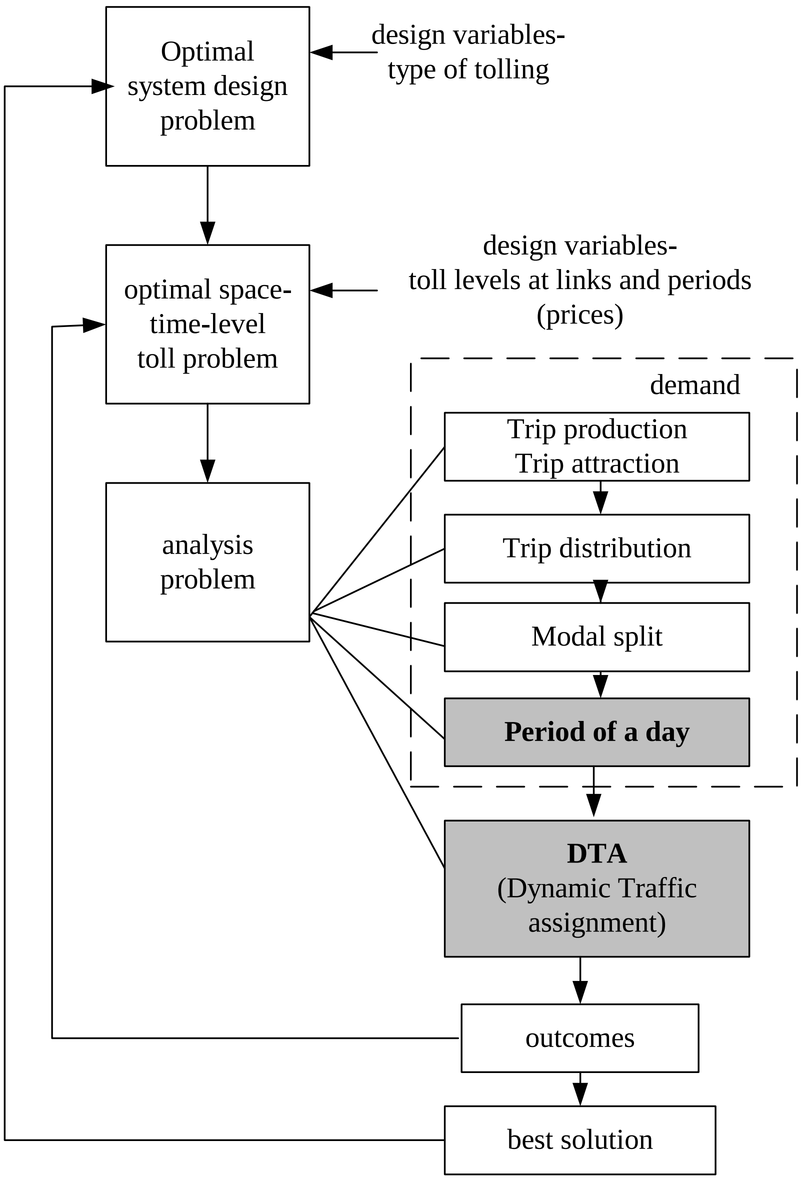

Figure 2.4: Characteristics of the optimal toll design problem

in the sense that they try to optimize personal objective functions within constraints. We discussed the objectives and constraints that an authority may adopt in tackling his tolling design problem. Given the analysis in this chapter we have chosen to adopt a mathematical programming approach as a tolling design methodology. The in-depth analysis of tolling regimes and design variables in a dynamic network context is the key contribution of this chapter that will be used in the operationalization of the design methodology in subsequent chapters.

The next chapters will further elaborate on the behavior of the involved actors (authority and travelers) by following a microscopic analysis using game theory. This will be followed by the derivation and formalization of the design optimization methodology at macroscopic level.

Part II

Micro-foundations of road pricing - a game theory approach

Chapter 3

Conceptual analysis of the road pricing problem - a game theory approach

3.1 Introduction

This chapter considers road pricing from its *microscopic-foundations*, meaning that interactions among individual actors are taken into account and analyzed. Motivation for using such a concept was to obtain a better understanding of the pricing phenomenon among policy makers stating that macroscopic results of pricing should be understood from their micro-foundations, that is the behavior of the individual actors. Schelling (1978) argues that macroscopic-phenomena should be examined from their micro-foundations, the behavior of individual actors. Ideas from microeconomic theory are applied to congestion in the work of Walters (1961) and Mohring (1970). This part of our research adopts this approach, aiming to build the simplest possible road pricing model that reflects individual behavior of the actors in road pricing (the road authority on the one side and travelers on the other).

After definition of the optimal toll design problem and research methodology to be used (see Chapter 2), it is necessary to formulate the *multi-actor* optimal toll pricing problem. The road pricing problem is a complex and controversial issue Verhoef et al. (1999) including different actors which influence each other in different ways. In order to have *more insight* into the nature of the optimal toll design problem (explained in Chapter 2), we will approach the road pricing problem considering the simplest case of pricing and network description. On the one side, the road authority, as one of the actors in road pricing problem, influence other actors (travelers) in their travel decision making. On the other side, travelers react on the influence of the road authority by changing their travel choices. From the behavioral point of view we are dealing with route and trip choice of travelers (the travelers have the opportunity to decide which route to take or to decide to stay at home and not take a trip) on a small hypothetical network.

The following questions arise and need to be answered in this chapter. Who is involved in decision-making in the road pricing problem and how should decisions be made? Who are the players, what are the interactions between them? Which strategies the players will play, and are their strategies opposite? What are the rules in the optimal toll design game? How to formulate the optimal toll design game? How to express different interactions (and roles) among players in the optimal toll design game?

In this chapter we propose a game-theory methodology to study the combined road pricing problem and travel choices. The interaction between road pricing and traveling is essentially the conflict between two types of players: a road authority and travelers. Travelers maximize their own individual travel utilities by choosing their travel decisions while the road authority maximizes an overall system objective (such as, e.g. rising revenues, social welfare, etc.) by setting tolls. This conflict among individual actors in the road pricing problem can be studied as a game among travelers and the road authority: the optimal toll design game.

Game theory has long been used to analyze individual's economic behavior. See e.g. Gibbons (1992) for a comprehensive review of game theory and its applications in economics. Since user's travel behavior is a subset of the general economic behavior, game theory can provide a powerful *analytical tool* to study the traffic assignment problem and its interaction with the optimal toll design problem. Game theory framework is used for modeling decision-making processes in which multiple interactive players are involved with different objectives, rules of the game, and assumptions. Considering the problem of designing optimal tolls on the network, there is a need for better insights into interactions among travelers and the road authority, their nature, and the consequences of these interactions. Game theory allows us to tackle this problem with some assumptions and to have a clearer picture about the optimal toll design problem.

The objective of travelers is to maximize their own individual travel utility while the road authority can have *different objectives* in order to reach different goals. For example, raising revenues or maximizing social benefit are only a subset of different policy objectives which can be considered in the optimal toll design game. Taking into account that *differentiation between travelers* is an essential issue in the road pricing problem, the heterogeneity of users is included in the game theory framework showing how different travelers will react differently on imposed tolls. Therefore, formulating the optimal toll design problem as an optimal toll design game using game theory notions will be the focus of this chapter .

The outline of this chapter is as follows. In Section 3.2 basic concepts and methodology of game theory is presented. A literature study of game theory applied to transportation problems is given in Section 3.3 including optimal traffic control problems and road pricing problems. Furthermore, in Section 3.4 game theory is applied to the road pricing problem where the players, strategies, rules and outcomes of the optimal toll design game are identified. A two-level game between the road authority and the travelers is identified. Problem definition of the optimal toll design game as a non-cooperative game as well as assumptions are given in Section 3.5. In Section 3.6 the optimal toll design game is mathematically formulated and the Nash principle is established as a main principle to solve the optimal toll design game. Different objectives of the road authority are presented and mathematically formulated in Section 3.7. Section 3.8 describes different game theory concepts such as monopoly game, Stackelberg and Cournot game applied to the road pricing problem. Different games between the travelers and the road authority are described and mathematically formulated. This chapter finishes with a conclusion part and further recommendations (Section 3.9). More about dynamic optimal toll design problem and inverse Stackelberg game can be found in a parallel study (Stankova et al. (2006)).

3.2 Basic concepts of game theory

In this section some basic game theory notions, needed for the formulation and solution (Chapter 4) of the optimal toll design game, are introduced and explained. For more comprehensive introduction to game theory, the interested reader is referred to e.g. Basar & Olsder (1995), Fudenberg & Tirole (1993).

3.2.1 Basic notions of a game

First, it is necessary to distinguish parts of the game and interaction between them. In game theory we can distinguish following parts of the game:

1. **Players** and their **interests** (interactive parties who participate in the game, subjects with their own objectives), **strategies** of the players (different alternatives to be chosen) and **payoffs** of the players (what they gain if play a certain strategy);
2. **Rules** of the game (who can do what and when);
3. **Outcomes** of the game (win or loss, payments between players, scores earned, etc.).

According to the three types of input to the game, game theory consists of three parts, see Ritzberger (2002): decision theory, representation theory and solution theory. **Decision theory** means the decision making between players according to the von Neumann-Morgenstern theory representing preferences of players over different strategies in a game. For more details, see Von Neumann & Morgenstern (1944). **Representation theory** describes the formal way to represent the rules of the game and all the strategies of the players. Rules of the game can be represented in two different ways: the *extensive (tree) form game* representation and the *normal (matrix) form game* representation. The extensive form game representation is more appropriate for multi-stage (dynamic) games while the normal game representation is more suited for one-stage (static) games. Static and dynamic games will be explained further in this section. For more comprehensive description of extensive and normal form game we refer to Ritzberger (2002). Both games will be illustrated and applied to the optimal toll design game in Chapter 4. **Solution theory** solves the game depending on what the players played and as a result the outcomes of the game are presented. Correlation between these cornerstones of game theory and the optimal toll design problem is given in Section 3.4.

3.2.2 Basic notions of different game types

Some game theory notions are very closely related to characteristics of the game itself. Therefore, different game types will be the focus of this subsection.

Regarding to the information that are available to players, a game can be a game with **complete information** (where all the players know all information about rules of the game and strategies of the players) and a game with **incomplete information** (where players have not complete information about rules of the game, etc.). Regarding to the outcome of the game, a game can be a **zero-sum game** (if the players payoffs sum to zero for all strategy combinations) or non zero-sum game.

There are two basic branches of games in game theory depending on the description of rules of the game: **cooperative** (rules are very broadly defined; they are not specific for every individual but for a group) and **non-cooperative** games (precise specification of the rules of the game for every individual player). Cooperative games can be used for description of coalitions of people and behavior of the group with the similar objective (e.g. political parties). The latter is the theory of games with complete rules and it enables an analysis of individual decisions by the players. In other words, the game is non-cooperative if each person involved follows its own interests which are partly conflicting with others. More about non-cooperative games can be found in Basar & Olsder (1995).

Regarding to the time dimension we can distinguish different kinds of games. **One-stage (static) games** means that the time dimension is not important hence all players make their decisions and react at the same time. Contrary to the one-stage (static) games, a **multi-stage (dynamic)** game means that the order in which decisions are made is important. Therefore, multi–stage (dynamic) games take into account the dynamic decision process evolving in time (time can be discrete or continuous), and with more than one decision maker. Each decision maker has his own utility (cost) function and access to different information. For more information, see Basar & Olsder (1995).

In addition, a game can be played in pure or mixed strategies. In **pure strategies,** each player is assumed to adopt only a single strategy, whereas in **mixed strategies**, the players are assumed to adopt probabilities for choosing each of the available strategies.

3.2.3 Basic notions of different game concepts

Regarding to the influence or the 'market power' which some players can have in a game, different game concepts can be distinguished. Distinguishing two different dimensions of 'power' in a game leads us to the following classifications. The first dimension refers to leadership versus Nash behavior in a game: does the player take the strategies (or actions) of the other players as given (Nash) or does (s)he, instead, take responses into account when evaluating the impacts of varying her own strategy (Stackelberg leadership)? The second dimension refers to the competitiveness of behavior: Is the player sufficiently ’big’ enough relative to the market to justify explicit consideration of the effects of his/her own actions upon prices (costs) given the other players’ strategies? The latter is normally referred to as ’marker power’, and it may exist both for a Nash player and for a Stackelberg leader.

In other words, with the ’market power’ we assume the influence of a particular player within a game against other players. For example, we suppose that one player can have full (complete), dominant (disproportional) or proportional (equal) market power (influence) with respect to the other player(s).

Regarding to different ’market power’ that one player can have against others, the following game concepts will be explained:

1. Monopoly (’social planner’) game;
2. Stackelberg (leader-follower(s)) game;
3. Cournot-Nash game.

1. A **monopoly** or **’social planner’** game is a ’solo player’ game that represents the best system performance for a given objective and thus may serve as a ‘benchmark’ solution to compare other solutions for the given policy objective. Actually, this game concept is not really a game because only one player is acting. In other words, this case will lead to a so-called *system optimal solution* of the game. This game solution shows what is best for the one player, regardless of the other players. From an economic point of view, it can be stated that in the monopoly game one player has the complete (or full) market power against other players.

2. A **Stackelberg game** represents so-called ’leader-follower’ game. One player starts the game (the leader) and the other(s) (followers) responds. From an economic point of view, in a Stackelberg game one player has *disproportional* (not equal) market power than others players in the game. The Stackelberg game is a two-stage (dynamic) game. For more details about Stackelberg games, see e.g. Ritzberger (2002).

3. A **Cournot-Nash game** is a game where all players have the same ’market power’. In contrast to the Stackelberg game, the players are now assumed to have a direct influence on the other player(s), having complete knowledge of the responses of the other player(s) to their decisions. This type of a so-called *duopoly game*, in which two players choose their strategies simultaneously and therefore one’s player’s response is unknown in advance to others, is known as a Cournot-Nash game. More information can be found in e.g. Hargreaves-Heap & Varoufakis (2004).

Table 3.1: Classification of games and solution methods

		complete information
static game	formulation	one-stage (static) game
	representation	normal form game
	method	dominated strategies
	solution	Cournot-Nash equilibrium
dynamic game	formulation	multi-stage (dynamic) game
	representation	extensive form game
	method	backward induction
	solution	Stackelberg model

3.2.4 Classification of games - an overview

In this research we restrict ourselves to the games based on complete information. As an overview, two groups of different classification of games can be made, as presented in Table 3.1. Two main classes of games are static games and dynamic games. Every group is further distinguished according to the following attributes:

1) formulation of the game (one-stage (static) or multi-stage (dynamic)),

2) representation of the rules of the game (normal form or extensive form),

3) method which is used for solving the game (dominated strategies, backward induction),

4) solution concepts to be assigned to the game (Cournot-Nash, Stackelberg, etc.).

From this table we can see that a Cournot-Nash game is a one–stage game, while Stackelberg game is a multi-stage game. Proposed game classification will be used in this chapter to formulate the optimal toll design game and in Chapter 4 to solve the optimal toll design game. More about normal-from and extensive-form game formulation can be found in Chapter 4.

3.3 Literature review of game theory applied to transportation problems

3.3.1 Transportation problems and game theory

Game theory is widely applied to solve different kinds of transportation problems. Game theory (Von Neumann & Morgenstern, 1944) presents an analytical approach to explain the choices of multiple interactive actors (players, agents) where every player in the game is trying to maximize his/her profit (assuming that players always behave rational in their decision making).

Game theory first appeared in solving transportation problems in the form of so-called *Wardropian equilibrium* of route choice, see Wardrop (1952), which is similar to the Nash equilibrium of an N-player game, see Nash (1950). The definition of a Nash equilibrium will be presented later in Section 3.6. Transportation problems, such as network design, traffic control, and road pricing have been studied from different modeling perspectives and under various assumptions using game theory approach. The literature study of both network design and optimal control problems will be mentioned in the next subsections. Furthermore, the literature study of game theory applied to the road pricing problem and heterogeneous users will be presented in the last subsections.

Game theory applied to Network Design Problems (NDP)

In the work of Fisk (1984) different problems are described for the first time in which a game theory approach is proposed as a potential source of solution algorithms for transportation systems modeling. In that paper, relationships are drawn between two game theory models (non-cooperative Nash and Stackelberg game). Examples of the Nash and Stackelberg games are explained in detail in the cases of carriers competing for intercity passenger travel and the signal optimization problem.

In Wie (1995) the dynamic mixed behavior traffic network equilibrium problem is formulated as a non-cooperative, N-person, non zero-sum dynamic game. A simple network is considered where two types of players (called user equilibrium (UE)-players and Cournot-Nash (CN)-players, respectively) interact through the congestion phenomenon. A UE-player (which represents a large number of homogenous travelers) attempts to minimize his/her individual transportation costs. In contrast, a CN player (e.g. a transportation company) attempts to minimize the total cost of transportation in the whole system. Each of the UE and CN players attempts to optimize his own objective by making simultaneous decisions of departure time choice, route choice, and departure flow rate over a fixed time interval.

In Bell (2000), the performance reliability of transport networks is studied using game theory approach. A two-player, non-cooperative game between the network user seeking a path to minimize the expected trip cost on the one hand, and an “evil entity” choosing link performance scenarios to maximize the expected trip cost on the other is established. At the Nash mixed-strategy equilibrium, the user is unable to reduce his expected trip cost by changing his path choice while the ‘evil entity’ is unable to increase this expected trip cost by changing the scenario probabilities, without cooperation.

An application of game theory to solve the risk-averse user equilibrium traffic assignment problem can be found in Bell & Cassir (2002). Network users have to make route choice decisions in the presence of uncertainty about route costs. Therefore, uncertainty requires network users to have a strategy towards risk. Network users “play through” all the possible states before selecting their best route. A deterministic user equilibrium traffic assignment is shown to be equivalent to the mixed strategy Nash equilibrium of N-player, non-cooperative game.

In Zhang, Peeta & Friesz (2005) a preliminary model of dynamic multi-layer infrastructure networks is presented in the form of a dynamic game. In particular, three network layers (car, urban freight and data) are modeled as Cournot-Nash dynamic agents. A dynamic game theoretic model of multi-layer infrastructure networks is introduced to solve the flow equilibrium and optimal budget allocation problem for these three layers. The assumption that a super authority oversees investments in the infrastructure of all three technologies is considered and thereby creates a dynamic Stackelberg leader-follower game.

Game theory applied to optimal traffic control problems

In Chen & Ben-Akiva (1998) the integrated traffic control and dynamic traffic assignment (DTA) problem is presented as a non-cooperative game between the traffic authority on the one side and highway users on the other. The objective of the combined control-assignment problem is to find dynamic system optimal signal settings and dynamic user-optimal traffic flows. The combined control-assignment problem is first formulated as a one-stage Cournot-Nash game: the traffic authority and the users choose their strategies simultaneously. Then, the combined problem is formulated as a two-stage Stackelberg game in which the traffic authority is the leader who determines the signal settings in anticipation of the users' responses. In the work of Garcia et al. (2000) a fictitious play for finding system optimal routings is studied.

In the work of Van Zuylen & Taale (2004) the integrated traffic control and traffic assignment problem is described as a three-player game (users and two road authorities). Two approaches (analytical and simulation) are used to analyze separate or integrated anticipatory control.

Road pricing problems and game theory

In Levinson (1998) the question is examined what happens when jurisdictions have the opportunity to establish tollbooths at the frontier. If one jurisdiction would be able to set his pricing strategy in a vacuum it is clearly advantageous to impose as high a toll on non-residents as can be supported. However, the neighboring jurisdiction can set a pricing strategy in response. This establishes the potential for a classical prisoner's dilemma consideration: in this case to tax (cooperate) or to toll (defect).

In Levinson (2003) an application of game theory and queuing analysis can be found to develop micro-formulations of congestion. Only departure time is analyzed in the context of a two and three-player game respectively where interactions among players affect the payoffs for other players in a systematic way.

The tax competition between countries that each maximize the surplus of local users is studied in De Borger et al. (2005). Three different pricing systems were considered: toll discrimination between local traffic and transit traffic; uniform tolls; and systems with tolls for local drivers only. For more information about governmental competition in road charging we refer to Ubbels (2006).

One study that does analyze a policy game between two different levels of governments with two different pricing policies in transport is a case study described in the European project MC-ICAM. In this analysis three different solutions have been considered: a centralized solution (pricing instruments are chosen simultaneously); a non -cooperative Nash equilibrium (each government takes the other's choice as given); and a Stackelberg equilibrium (the region acts as leader and chooses its policy instrument first). More information about this project can be found in MC-ICAM (2004).

3.3.2 Heterogeneous users

In previous studies it is assumed that travelers are homogenous and that they behave on the same way. Nevertheless, users differ in their preferences where and when to travel. Considering the behavior of these different user classes is essential in the road-pricing problem in that different users (travelers) will behave in different ways. Recently, the issue of heterogeneous users has gained a lot of attention in the scientific community showing that it is an important problem.

Difference between users can be formulated regarding to different criteria (vehicle types, value of time, etc.). More about differences in value of time can be found in e.g. Verhoef & Small (2004) and Yang et al. (2001). Wang et al. (2004) studied a monopoly situation on a private highway, involving strategic interactions between the highway and transit operators. Heterogeneity of travelers is taken into account by considering a continuous distribution of values of time of travelers.

From the literature study, we can conclude that there is a lack in the literature considering different user groups in the optimal toll design problem using game theory approach. In addition, the heterogeneity of users is essential in the road pricing problem. Furthermore, there is a lack in the literature about different policies of the road authority and outcomes that can be result of the different objectives and games played by the travelers. Also, it should be noted that all authors considered fixed demand in their approaches. Therefore, different user groups in the optimal toll design problem as well as different game concepts in the elastic demand framework with route choice will be the focus of this research. Furthermore, in this chapter we examine the road pricing problem from its micro-foundations, that is the behavior of individual actors.

3.4 Game theory concepts applied to the optimal toll design problem with heterogeneous users

Formulating and solving the optimal toll design problem using a game theory approach needs a definition of basic decision theory cornerstones of the problem. Before formu-

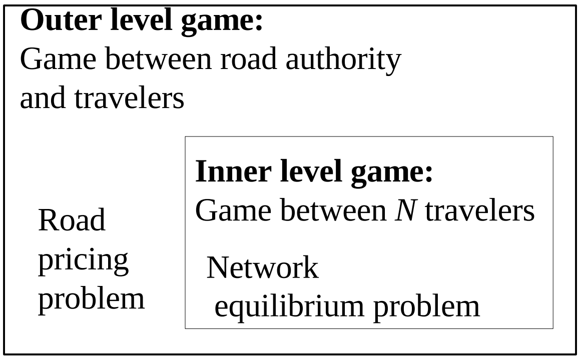

Figure 3.1: Two-level optimal toll game

lating the optimal toll problem using game theory notions it is useful to highlight that *subjective payoff functions* will be used as criteria for travel behavior of travelers. We suppose that the travelers will decide which route to take and to take a trip or not depending on their *individual utility functions*. For this purpose the *utility maximization theory* is used to describe the travel behavior. Also we will restrict ourselves to apply non-cooperative game theory meaning that the travelers will not cooperate with each other (nor with the road authority) but they will play as individual players. In the optimal toll problem travelers can take different routes or decide to travel or not while the road authority sets different values for tolls. Furthermore, our aim is to apply game theory to the optimal toll design problem including heterogeneous users.

In fact, there are two games played in conjunction with each other. The first game (*inner level game*) is a non-cooperative game where all N travelers aim to maximize their own utility by choosing the best travel strategy (i.e. trip choice and route choice), taking into account all other travelers' strategies (the inner level in Figure 3.1). The second game (*outer level game*) is a game between the travelers and the road authority, where the road manager aims to maximize some network performance by choosing a toll strategy, taking into account that travelers respond to the toll strategy by adapting their travel strategies (combination of the lower and upper level in Figure 3.1).

The outer and inner game form a so-called *bi-level game,* a game which consists of two different, mutually interrelated problems. The outcome of the optimal toll design game is then the combined solution of both games. This bi-level problem structure for more complex cases will be explained more in Chapter 5 (in the form of so-called bi-level optimization problems).

The correlation between non-cooperative game theory notions and the optimal toll design problem is illustrated in Figure 3.2. The three building blocks in game theory (decision theory, representation theory and solution theory) are represented by the following elements: 1) players, strategies and payoffs, 2) rules of the game, and 3) outcomes of the game. The main elements (three building blocks) of the optimal toll design game are: 1) road authority and travelers are players in the game; strategies are route and trip choices; and payoffs are toll levels; 2) rules of the game are change of behavior in order to max-

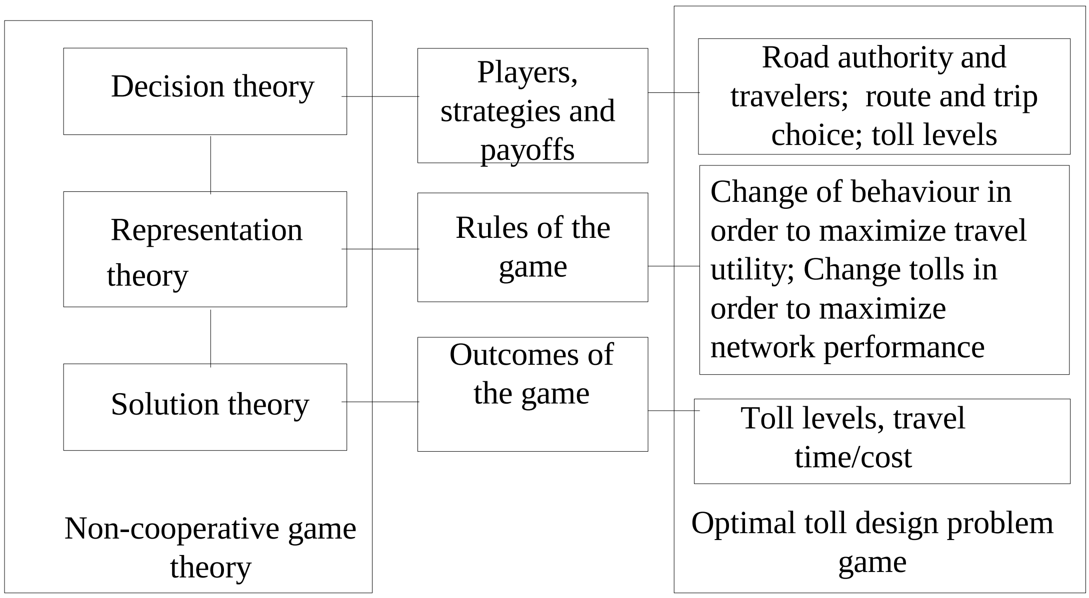

Figure 3.2: Three building blocks for solving games using game theory

imize travel utility or change tolls in order to maximize network performance; and 3) outcomes of the optimal toll design game are toll levels and travel time/cost.

The outer level game, being the **road pricing problem,** consists of the following elements:

- Players: the government (the road authority) on the one side and *N* potential travelers on the other;
- Strategies of players: a) toll levels (for the government), b) travel choices such as route and trip choice (for travelers);
- Payoff functions of players: a) payoff for the government (e.g. social welfare, revenues), b) payoff for the travelers (travel utilities);
- Rules of the game: the government sets the tolls taking the travelers' behavior and responses into account in order to optimize a certain objective.

The inner level game, being the **network equilibrium problem**, consists of the following elements:

- Players: *N* travelers;
- Strategies of players: travel choices such as route and trip choice;
- Payoff functions of players: travel utilities;

- Rules of the game: travelers make their optimal trip and route choice decisions as to maximize their individual subjective utilities given a specific toll pattern over space.

It should be noted that we described the network equilibrium problem, as a game where individuals have 'market power', hence generalized (route) prices depend significantly on their own decisions. Our motivation was to investigate the individual interactions between travelers. However, the essence of traffic congestion in practise is that there are many travelers involved (and market power then evaporates when the number of travelers increases). For that purpose, the extension to many travelers was necessary (see Part III, Chapter 5 and 6).

The inner level game, i.e. the network equilibrium problem, has been subject of many studies. For more information, see e.g. Wie (1995). Therefore, our main focus is to investigate the outer level game between the road authority and users, although the inner level game between travelers is part of it. Every player should choose a strategy in order to achieve his/her objective. What actually will be done depends on quantities not yet known and not controlled by the decision maker. The decision maker cannot influence the course of the events once he/she has fixed his/her strategy.

3.5 Problem definition of the optimal toll design problem as a non-cooperative game and assumptions

The interactions between the travelers and the road authority can be seen as a non-cooperative, non-zero sum, *(N+1)* player game between a single traffic authority on the one side and N users (travelers) on the other. N travelers are divided into a number of groups depending on their value of time (VOT). Whereas the interest of travelers is to maximize utility of traveling, the interest of the road authority may be to raise revenues or to minimize the total travel time on the whole network, etc. The objective of the road-pricing problem, which is the combined optimal toll design and user-optimal traffic assignment problem, is to find system-optimal tolls and corresponding user-optimal traffic flows. This road-pricing problem is an example of a *two-stage game*. The user-equilibrium traffic assignment problem (lower level problem) can be formulated as non-cooperative, N-person, non-zero sum game solved as a Nash game. The upper level problem may have different objectives depending on what the road authority would like to achieve. A conceptual framework for the optimal toll design problem with route choice and elastic demand is given in Figure 3.3.

The road authority sets tolls on the network in order to optimize his objective while travelers respond to tolls by adapting their travel decisions. Depending on travel costs, they can, e.g. decide to change routes or decide not to travel at all. In the road-pricing problem, we are dealing with an N+1-player game, where there are N players (travelers) making

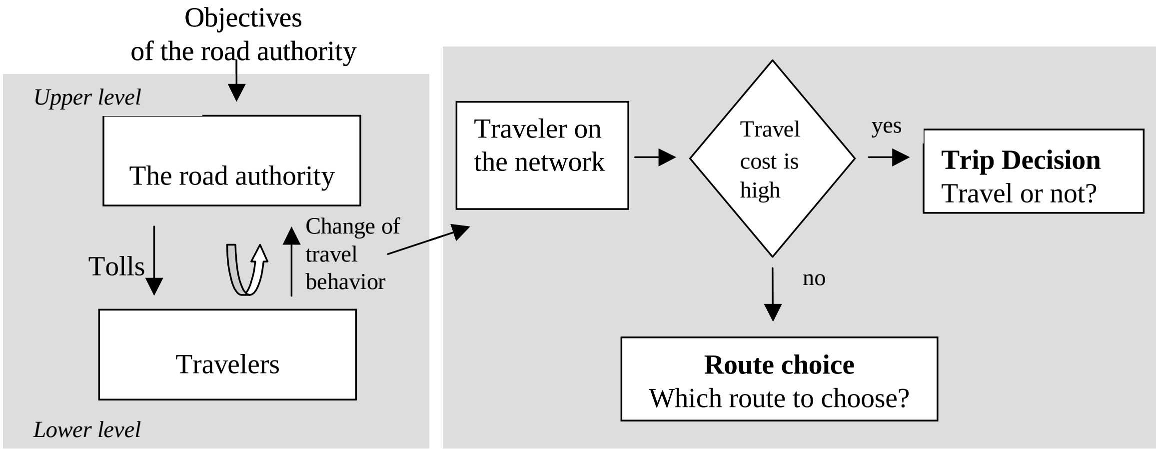

Figure 3.3: Conceptual framework for the optimal toll design problem with trip and route choice

travel choice decisions, and one player (the road manager) making a design decision (in this case, setting road tolls). Adding the traffic authority to the game is not as simple as extending an N-player game to an N+1 player game, because the strategy space and the payoff function for this additional player differs from the rest of the N players.

Some *assumptions* of the analysis should be given here. First, we assume that each player is a *rational decision-maker* (each player in the road-pricing game is trying to maximize his/her own profit). Secondly, we assume *common knowledge* (or complete information) meaning that every player knows that every other player is rational and each player knows that everybody knows, and so on *ad infinitum*. Thirdly, we assume that all games are games with *perfect information*. A game with perfect information means that each decision maker, whenever he/she has to take a decision, knows the entire history of the game (the players have precisely the same information as an outside observer). In other words, for the road pricing game we assume that all information about travel attributes (toll levels, available routes, travel times) are known to all travelers as well as the strategies, rules and outcomes of the game.

3.6 Model formulation of the optimal toll design game

In this section the model formulation for both the network equilibrium problem and the optimal toll design game will be given. Since the purpose of this section is to gain more insight into the *structure of the optimal toll design problem* with different user classes using game theory, we restrict ourselves to the case of a simple network in which only one origin-destination (OD) pair is considered (see Figure 3.4).

In this section we use, among others, the travel utility for individuals as the objective to maximize for travelers. Between this OD pair, different non-overlapping path alternatives are available. Each traveler i chooses which path p to take and whether to travel or not. In this work we will use the notation of Altman et al. (2003) adapted for the road-pricing

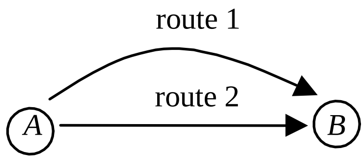

Figure 3.4: Simple network with a single OD pair

problem. The generalized path travel cost function of traveler i for path p, includes the travel time costs and the toll costs, see Equation (3.1).

$$c_{ip} = \alpha_i \tau_p + \theta_p \tag{3.1}$$

where τ_p is the travel time of path p, θ_p is the toll costs of path p, and α_i denotes the value of time (VOT) for user i which converts the travel time into monetary costs. Tolls are chosen to be constant in time. Let U_{ip} denote the trip utility of traveler i for making a trip along path p. This trip utility, U_{ip} (see Equation (3.2)) consists of a fixed utility gained by making the trip, (or arriving at the destination) $\overline{U}$, and a disutility consisting of the generalized path travel costs, c_{ip},

$$U_{ip} = \overline{U} - c_{ip}. \tag{3.2}$$

According to *utility maximization theory*, a trip will be made only if the utility of doing an activity at a destination minus the utility of staying at home and the disutility of traveling is positive. In other words, if $U_{ip} \leq 0$ then no trip will be made. By including a *fictitious route* in the route choice set (see 'route 3' in Figure 3.5) representing the travelers' choice not to travel, and attaching a utility of zero to this 'route' alternative, we combine route choice and trip choice into the model. Travelers are assumed to respond according to *Wardrop's equilibrium law* extended with elastic demand:

Definition 1 *At equilibrium, no user can improve his/her trip utility by unilaterally making a different route choice or trip choice decision (switching route or activity).*

It should be mentioned that we only take into account the deterministic case, which means that we limit ourselves to *pure strategies* (in mixed strategies a certain probability is attached to each strategy, representing the stochastic case). In the following subsections the network equilibrium problem and toll design problem will be explain using game theory notions.

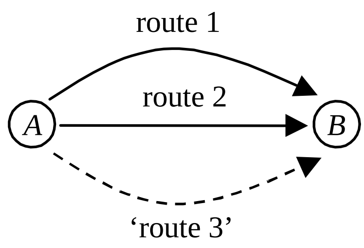

Figure 3.5: Simple network with a fictitious route

3.6.1 Inner level game: Network equilibrium problem

Let us first consider the N-player game of the travelers, where $\overline{S}_i$ is the set of available alternatives for traveler i, $i \in \{1, ..., N\}$. These alternatives consist of the set of available routes (including the fictitious route for not making a trip). Each traveler i can choose a strategy (route) $s_i \in \overline{S}_i$. The optimal strategy $s_i^* \in \overline{S}_i$ that traveler i will follow depends on the toll strategy set by the road manager, denoted by vector θ and on the strategies of all other travelers, denoted by $s_{-i} = (s_{1,}..., s_{i-1}, s_{i+1}, ..., s_{N).}$ We assume that each traveler decides independently and seeks the maximum utility payoff, taking into account the possible choices of the other travelers. Let $J_i(s_i(\theta), s_{-i}(\theta), \theta)$ denote the utility payoff for traveler i for a given tolling strategy. The utility payoff can include all kinds of travel utilities and travel costs. The utility payoff for traveler i can be expressed as the utility stated in Equation (3.3),

$$J_i(s_i(\theta), s_{-i}(\theta), \theta) = U_{is_i} \tag{3.3}$$

Note that s_i in U_{is_i} (Equation 3.3) denotes a specific path p (including the fictitious route) in Equation (3.2). Therefore, chosen strategy s_i (which is actually chosen alternative p) determines the trip utility U_{ip} of traveler i if he has chosen route p. As shown in Equation (3.3) the trip utility, U_{is_i} depends not only on what the traveler i chooses, but also on decisions of all other travelers (e.g., if all travelers on the network choose the same route p, than congestion on that route will arise having effects on the travel costs of all travelers). Utility U_{is_i} depends also on tolls θ which are set on the network.

Definition 2 *If all other travelers play strategies s_{-i}^*, then traveler i will play the strategy that maximizes his payoff utility, i.e.*

$$s_i^*(\theta) = \arg\max_{s_i \in \bar{S}_i} J_i(s_i(\theta), s_{-i}^*(\theta), \theta) \tag{3.4}$$

If Equation (3.4) holds for all travelers $i \in \{1, ..., N\}$, then $s^*(\theta) = (s_i^*(\theta), s_{-i}^*(\theta))$ is called a Nash equilibrium for the inner level game given the control strategy θ.

In this equilibrium, no traveler can improve his utility payoff by unilateral deviation. In other words, a Nash equilibrium is a set of strategies such that no player can improve by changing strategy given that other players keep their strategies fixed. This corresponds very closely with the independently developed Wardrop (1952) user equilibrium principle used in route choice. In the following sections, the optimal toll design game is formulated using the Nash principle within different game concepts.

3.6.2 Outer level game: toll design problem

Let us now consider the complete N+1-player game where the road manager faces the N travelers. The set $\overline{\Theta}$ describes the alternative tolling strategies available to the road manager. Suppose he chooses strategy $\theta \in \overline{\Theta}$. Depending on this strategy and on the strategies chosen by the travelers, $s^*(\theta)$, his utility payoff is denoted by $\overline{R}(s^*(\theta), \theta)$ and represents e.g. the total system utility or the total profits made.

Definition 3 *The road manager chooses the strategy* θ^* *with which he aims to maximize his utility payoff, depending on the responses of the travelers:*

$$\theta^* = \arg \max_{\theta \in \bar{\Theta}} \overline{R}(s_i^*(\theta), \theta) \tag{3.5}$$

If Equations (3.4) and (3.5) are satisfied for all (N+1) players, where $\theta = \theta^*$ *in Equation (3.4), then a Nash equilibrium for the outer level game is reached, in which no player can be better off by unilaterally playing another strategy.*

Although all equilibria use the Nash concept, different equilibria or game types can be defined in the N+1-player game depending on the influence each of the players has in the game.

3.7 Different objectives of the road authority in the optimal toll design problem

The objectives of the road authority and the travelers are different and sometimes even opposite. The upper level objective may be to minimize total system travel time, to relieve congestion, to improve safety, to raise revenue, to improve total system utility, etc. Which objective the road authority will apply will have a different influence on the optimal toll levels. We can also say that the objective of the road authority is a *system optimum solution,* while the objective of the travelers is to reach a *user equilibrium solution.* Depending on the authority's objective, different payoff functions for the road authority can be formulated.

1) In the case the road authority aims at **maximizing total toll revenues**, the following payoff function may be used:

$$\overline{R}(s^*(\theta), \theta) = \sum_p q_p(s^*(\theta))\theta_p \tag{3.6}$$

where $q_p(s^*)$ denotes the number of travelers using route p, which can be derived from the optimal strategies s^*. Clearly, setting tolls equal to zero does not yield any revenues, while setting very high tolls will make more travelers decide to travel on another route or decide to not to travel at all. Hence, there will clearly be an optimum toll level somewhere in between low and high tolls.

2) Assuming that the road authority's objective is **maximizing total travel utility** (the utility of all network users together), the objective is defined as the sum of the payoff values of all travelers (including the tolls):

$$\overline{R}(s^*(\theta), \theta) = \sum_{i=1}^{N} J_i(s^*(\theta)) \tag{3.7}$$

This policy objective would be interesting if the government is concerned about the travelers and would like to do the best for travelers.

3) The social surplus can be determined by adding the toll revenues to the total trip utilities, such that the following objective can be used to **maximize social surplus**:

$$\overline{R}(s^*(\theta), \theta) = \sum_{i=1}^{N} J_i(s^*(\theta)) + \sum_{p} q_p(s^*(\theta))\theta_p \tag{3.8}$$

The above-mentioned objective is the combination between two previously mentioned objectives: total travel utility and revenues. The social surplus as an objective for the road authority can be seen as a benefit for whole society, thus for travelers as well as for the road authority. In Equation (3.8) the first term captures the benefit of travelers and the second the benefit of the road authority.

The formulated policy objectives will be used in Chapter 4 to illustrate the optimal toll design game and different outcomes. The previously stated policy objectives of the road authority are only a subset of possible objectives to be considered.

3.8 Different game concepts applied to the optimal toll design problem

Our topic in this section is to formulate and solve *one-stage* (*static)* optimal toll design game with complete information using a *normal form game* and the *method of dominated strategies* applying the *Nash equilibrium principle*. Monopoly and Cournot game are also examples of this game formulation. On the other hand, we formulate the optimal toll design game as a two-stage game with complete information where we used an *extensive game* representation and *backward induction model* and applied *Stackelberg solution* (see Table 3.1). More detailed description of the games can be found in Ritzberger (2002).

In this section we will explain and mathematically formulate one-stage Monopoly or 'social planner' game, Cournot optimal toll design game, and two-stage Stackelberg optimal toll design game. These game formulations will be used in next Chapter 4 to solve the optimal toll design game for different policy objectives and heterogeneous users.

3.8.1 Monopoly game ('social planner' game)

In this case, the road authority not only sets its own control, but also controls the strategies that travelers have. In this game concept, the road authority is acting as a pure monopolist. This game solution shows what is best for the one player (in this case the road manager), regardless of the other players (the travelers). However, a monopoly solution may not be realistic since it is usually not in the users' best interest and it is practically difficult to force travelers choosing a specific route without an incentive. From an economic point of view, it can be stated that in the monopoly game the road authority has complete (or full) market power and serves as a 'benchmark' solution. This problem can be mathematically formulated as follows.

Definition 4 *If a pair* (s, θ) *where s is the traveler's strategy (*$s \in \overline{S}$*) and* θ *is the strategy for the road authority (*$\theta \in \bar{\Theta}$*), satisfy the following expression*

$$(s^*, \theta^*) = \arg \max_{\theta \in \bar{\Theta}, s \in \bar{S}} \overline{R}(s, \theta) \tag{3.9}$$

then the pair (s^*, θ^*) *is known as Monopoly ('social planner') solution which maximizes the payoff of the road authority,* $\overline{R}(s, \theta)$*, where* s^* *is the optimal traveler's strategy and* θ^* *is the optimal strategy of the road authority.*

In other words, the road authority will maximize its payoff utility $\overline{R}(s, \theta)$ by setting toll θ^* and route s^*.It should be mentioned that this policy objective will be used as a reference case for other game concepts, because of its limited practical relevance. Namely, in this game concept the road authority 'physically' control the actions of the travelers, and set toll independent of their actions. While this is possible for a two-players case (see Chapter 4), on a 'real' road network with flows of hundreds (or more) of vehicles per hour, this seems impossible.

3.8.2 Stackelberg game

In this case, the road authority is the 'leader' by setting the toll, thereby directly influencing the travelers that are considered to be 'followers'. The travelers may only indirectly influence the road authority by making travel decisions based on the toll. It is assumed that the road authority has *complete knowledge* of how travelers respond to control measures. The road authority sets the toll and the travelers follow by playing. From an economic point of view, in a Stackelberg game one player has *disproportional* (not equal) market power than others players in the game. In this case the road authority has more market power than the travelers. The Stackelberg game is a two-stage (dynamic) game. For more details about Stackelberg games, see e.g. Ritzberger (2002).

The equilibrium solutions can be determined by the *backward induction* method where the traffic authority initiates the moves by setting a toll strategy. For a more detailed explanation about the backward induction method in game theory, see e.g. Basar & Olsder (1995). An illustration of steps for the optimal toll design game are as follows:

1. The road authority chooses toll values from the feasible set of tolls;
2. The travelers react on the route cost (with tolls included) by adapting their route and/or trip choice;
3. Payoffs for the road authority as well as travelers are computed;
4. The optimal strategy for the road authority including the strategies of travelers is chosen.

The problem can be mathematically formulated as follows.

Definition 5 *If a pair (s, θ) satisfies the following expression*

$$\theta^* = \arg\max_{\theta \in \bar{\Theta}} \overline{R}(s_i^*(\theta), s_{-i}^*(\theta), \theta) \tag{3.10}$$

where

$$s_i^* = \arg\max_{s_i \in \bar{S}_i} J_i(s_i, s_{-i}^*, \theta) \qquad, \forall i = 1, ..., N \tag{3.11}$$

then the pair (s^*, θ^*) is known as Stackelberg solution where s^* is the optimal strategy of the traveler (maximizing the payoff of traveler, (J)) and θ^* is the optimal strategy of the road authority (maximizing the payoff function of the road authority, $(\overline{R})$).

In other words, the road authority optimize its own objective maximizing the payoff functions yielding at the optimal toll value while the travelers optimize their objective yielding at the optimal route choice.

In order to solve the Stackelberg game it is necessary to solve the Nash equilibrium for the travelers if the road authority sets toll values. The expression (3.11) is a Nash solution for the travelers (inner optimal toll design game) while the expression (3.10) is the objective of the road manager (outer optimal toll design game).

3.8.3 Cournot game

In contrast to the Stackelberg game, the travelers are now assumed to have a direct influence on the road authority, having complete knowledge of the responses of the road authority to their travel decisions. The road authority sets tolls depending on the travelers' strategies. This type of a so-called *duopoly game*, in which two players choose their strategies simultaneously and therefore one's player's response is unknown in advance to others, is known as a Cournot game. For more information about Cournot, see more in e.g. Hargreaves-Heap & Varoufakis (2004). Mathematically the problem can be formulated as follows.

Definition 6 *If a pair* (s, θ) *satisfies the following expressions*

$$\theta^* = \arg\max_{\theta \in \bar{\Theta}} \overline{R}(s_i^*, s_{-i}^*, \theta) \tag{3.12}$$

and

$$s_i^* = \arg\max_{s_i \in \bar{S}_i} J_i(s_i, s_{-i}^*, \theta^*) \qquad , \forall i = 1, ..., N \tag{3.13}$$

then the pair (s^*, θ^*) *is known as Cournot solution where* s^* *is the optimal strategy of the traveler (with* θ^**) and* θ^* *is the optimal strategy of the road authority (maximizing the payoff function of the road authority,* $(\overline{R})$*).*

In order to solve a Cournot game it is necessary to consider travelers and the road authority *simultaneously* and to determine the Nash equilibrium solution for both. From the economical point of view it should be noted that in this game concept travelers as well as the road authority have equal market power. The motivation to consider this game concept is to investigate the (individual) interactions between travelers and road authority. In addition, the aim is to compare the outcomes with other game concepts and to prove which game concept is the most realistic.

Considering the nature and interactions between individual actors in the optimal toll design problem a discussion about which is the most realistic game concept can be held (see Figure 3.6).

Taking into account the 'market power' that players have in the optimal toll design game, and the nature of the road pricing itself (see Chapter 2) one can conclude that the most realistic situation is that the road authority has more 'market power' than travelers, the Stackelberg game concept. The situation where the road authority has full 'market power' we already characterized as a 'benchmark' solution, the situation which will never happen in reality. Finally, the situation where the road authority and travelers have the same 'market power' (and road authority sets tolls taking behavioral decisions of travelers as given) is also not expected to happen in reality. Therefore, one can conclude that the game concept where the road authority is the 'leader' and travelers are 'followers' is the most realistic situation in the optimal toll design problem. In further work and in experiments (Chapter 4) we will mostly consider this game concept as the most realistic one.

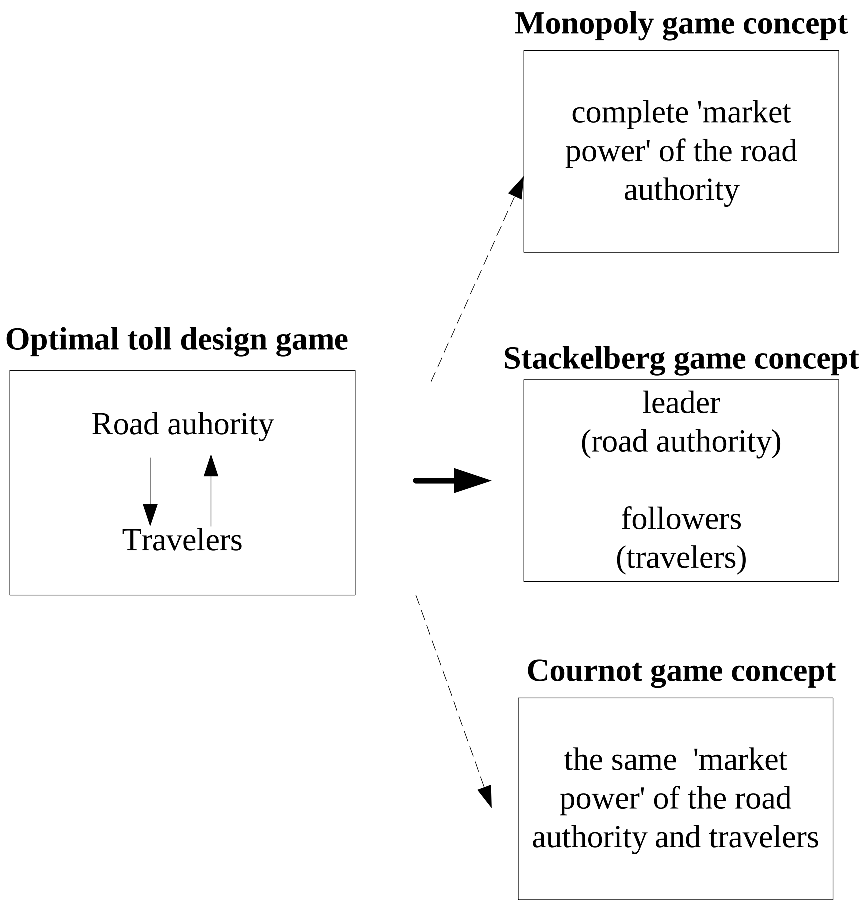

Figure 3.6: Mapping between the optimal toll design game and different game concepts

3.9 Summary and Conclusions

The purpose of this chapter was to gain more insight into the road-pricing problem using concepts from game theory as well as different policy objectives and heterogeneous users. In this chapter we used game theory methodology to integrate optimal toll design problem and traffic assignment. We use the game theory approach because it provides a framework for modeling a decision-making process in which multiple players are involved.

A literature review of game theory applied to transportation problems is given showing a lack in literature concerning optimal toll design problem with heterogeneous users and different policy objectives. The cornerstones for formulating and solving the optimal toll problem using game theory are described and applied on the optimal toll design problem. The classification of the optimal toll design problem with respect to the information that the players have, possible representation of the optimal toll problem (as a normal or extensive form) are given.

We focus on the optimal toll design game more than on the network equilibrium game. The Nash game formulation for the road pricing problem, as well as mathematical formulation of the optimal toll design problem resulted from this chapter. A Nash principle is established for bi-level optimal toll design problem and different game concepts are formulated. The dynamic optimal toll design problem is studied in Stankova et al. (2006) where the inverse Stackelberg game is introduced.

More specifically, the new results from this chapter are:

- formulation of the optimal toll design game using game theory notions, in terms of players, strategies payoffs, as well as rules and outcomes of the game;
- the payoff of travelers as well as the road authority is expressed using utility functions;
- the optimal toll design game is mathematically formulated using different game concepts (one-stage and two-stage game);
- different policy objectives of the road authority are identified and mathematically formulated.

The following intuitive conclusions can be derived from this chapter:

- depending on the market power between the road authority and travelers, different optimal toll design games can be proposed;
- taking all game concepts into account, the two-stage optimal toll design game seems to be the most realistic game because we expect that the road authority will influence the travel behavior of travelers and have more 'market power' than travelers;

The solution of the optimal toll design game using proposed game concepts will be the subject of the next chapter. The theory presented here can be extended to include other relevant travel choices such as e.g. departure time choice as well as to include heterogeneous travelers and imperfect information on the part of the road users. An extension can be to apply the proposed game-theory framework for larger cases (e.g. for large number of players or on a larger network). For practical use, the presented game-theoretic analysis should be translated into a modeling system with which tolling designs for real-size road networks become feasible. For that purpose, the bi-level optimization framework will be used in this research (see Chapter 5).

Chapter 4

Solving the optimal toll design game using game theory - a few experiments

4.1 Introduction

Using game theory notions we investigated a new approach to formulate the optimal toll design problem (see Chapter 3) with a focus on different policy objectives of the road authority and heterogenous users. The aim is to gain more insight into determining optimal tolls as well as into the behavior of users after tolls have been imposed on the network. As stated in the previous chapter, the problem of determining optimal tolls is defined using utility maximization theory. Different game concepts (Cournot, Stackelberg and monopoly game) are mathematically formulated and relationships between players, their payoff functions and rules of the games are defined.

In this chapter different game concepts (proposed in Chapter 3) are illustrated and solved for different objectives of the road authority. We focus on giving more insights into the following questions. How will the travelers change their travel behavior after introducing road pricing? How will travelers interact with each other and how can the road authority influence or even control travel behavior of travelers? How will different user groups react on different tolls, imposed by the road authority and how will they change their travel behavior? Which pricing strategies (toll values) should the road authority apply in order to achieve its desired objective?

These questions are answered using different game theory concepts described in the previous chapter. In this chapter different policy objectives are analyzed showing how interactions between players result in different outcomes (for travelers as well as for the road authority) of the optimal toll design games. A few experiments are done illustrating outcomes of different games and different scenarios. First, payoff functions and tables are established for travelers and the road authority. After that different game concepts are applied. A Monopoly game in which the road authority is a solo player is demonstrated showing the 'benchmark' solution of the optimal toll design game. The 'benchmark' solution means that that game solution can be used as a standard that other game solutions can be compared with. The Monopoly game is chosen as a ”benchmark” solution because it represents the best solution that can happen from the system point of view, but not in reality. Using the Stackelberg game the concept where the road authority is the leader and travelers are followers is analyzed. Finally, the situation where the road authority and travelers have the same market power is presented in the form of a Cournot game.

In order to analyze the behavior among individual actors we will analyze the simplest pricing problem. Nevertheless, results from this chapter show different outcomes both in terms of optimal tolls as well as in payoffs for travelers. It is shown how different groups of travelers (players) will react differently (change their travel behavior) on travel conditions depending on their value of time. The results of all three different game concepts are compared and the optimal toll values and the strategies for players are presented. The most realistic game concept is Stackelberg game where the road authority has more 'market power' (as a leader) than other players (travelers or followers). There exist multiple optimal solutions and the objective function may have a non-continuous shape (because of pure strategies for players chosen in experiments).

In the first section of this chapter a network description for the experiments is given as well as payoff functions of the travelers and travel time functions. The case study of maximizing total travel utility is presented in Section 4.3 with analysis of different game concepts. In Section 4.4 the policy objective of maximizing revenues is presented. In Section 4.5 the objective of maximizing of social surplus is analyzed. A comparison among different policy objectives with regard to the Stackelberg game is given in Section 4.6. A case study with heterogenous users are considered in Section 4.7. The chapter concludes with a summary and an overview of the most important aspects of the presented approach (Section 4.8).

4.2 A few experiments including different policy objectives

In this section the set-up for the experiments (utility functions for the players, payoff tables, network description, etc.) is outlined. A necessary definitions of a bi-matrix game, Nash equilibrium solution and dominance which will be used in this chapter, are given in this section.

In the following, a *general bi-matrix game (*see more in Basar & Olsder (1995)*)* will be used for solving different game theory concepts (Stackelberg game, Cournot game, respectively). Again, J_i is the utility payoff of traveler i while J_j is the utility payoff of traveler j. Let s_i denotes the strategy of the traveler i and s_j the strategy of traveler j.

Definition 7 *A bi-matrix game is comprised of two* $(m \times n)$ *dimensional matrices* $J_i(s_i, s_j)$ *and* $J_j(s_i, s_j)$*, where each entry (pair of matrix elements)* s_i *and* s_j *denotes the outcomes of the game corresponding to a particular pair of decisions made by each of the two players.*

Because $m = |s_i|$ and $n = |s_j|$, the game is comprised of one $m \times n$ dimensional matrix. In the sake of simplicity we will present these two separate matrices J_i and J_j in one matrix X, where each element in matrix X has two components, one of matrix J_i and another of matrix J_j.

Definition 8 *A pair of strategies (s_i^*, s_j^*) is said to constitute a non-cooperative Nash equilibrium solution to a bi-matrix game if the following pair of inequalities is satisfied:*

$$J_i(s_i^*, s_j^*) \geq J_i(s_i, s_j^*) \qquad \forall s_i \in \overline{S}_i \tag{4.1}$$

and

$$J_j(s_i^*, s_j^*) \geq J_j(s_i^*, s_j) \qquad \forall s_j \in \overline{S}_j \tag{4.2}$$

A game can admit multiple Nash equilibrium solutions, with the equilibrium payoffs being different in each case. This raises the question of whether it would be possible to order different Nash equilibrium solutions so as to declare only one of them as the most favorable equilibrium solution. However, this is not always possible, since a total ordering does not always exist between pairs of numbers. But a notion of '*dominance*' can be introduced through a partial ordering.

Definition 9 *A pair of strategies (s_{i_1}, s_{j_1}) is said to be dominant over (or better than) another pair of strategies (s_{i_2}, s_{j_2}) if*

$$J_i(s_{i_1}, s_{j_1}) \geq J_i(s_{i_2}, s_{j_2}) \tag{4.3}$$

and

$$J_j(s_{i_1}, s_{j_1}) \geq J_j(s_{i_2}, s_{j_2}) \tag{4.4}$$

and if at least one of these inequalities is strict. A Nash equilibrium strategy is said to be the admissible strategy if there exists no better (more dominant) Nash equilibrium strategy pairs (see more in Basar & Olsder (1995)).

Let us consider the following simple problem to illustrate how the road-pricing problem can be analyzed from a micro-economic perspective using a game theory approach. Suppose there are two individual actors who want to travel from A to B. There are two alternative routes available to go from A to B. The first route is tolled (toll is equal to θ) while the second route is untolled. Depending on the travel utility (including tolls), the travelers may decide to take either route 1 or route 2, or not to travel at all. The latter choice is represented by a third *virtual (fictitious)* route, such that we can consider three route alternatives as available strategies to each traveler, i.e. $S_i = \{1, 2, 3\}$ for traveler $i = 1, 2$. Figure 4.1 illustrates the problem. Each strategy of travelers yields a different payoff, depending on the utility of the trip, the travel time on the route (that increases whenever more travelers use it) and a possible route toll.

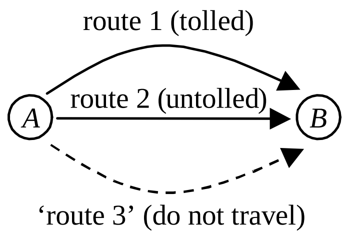

Figure 4.1: Network description

We assumed that congestion will appear only if both travelers choose the same route for traveling from A to B. The route travel times are given as a function of the chosen strategies in the sense that the more travelers use a certain route, the higher the travel time (see Equations (4.5) and (4.6)):

$$\tau_1\left(s_1(\theta), s_2(\theta)\right) = \left\{ \begin{array}{llll} 10, & \text{if} & s_1(\theta)=1 \ \ \text{or} \ \ s_2(\theta)=1 & \text{(e.g. flow on route 1 is 1)} \\ 18, & \text{if} & s_1(\theta)=1 \ \ \text{and} \ \ s_2(\theta)=1 & \text{(e.g. flow on route 1 is 2)} \end{array} \right\} \tag{4.5}$$

and

$$\tau_2\left(s_1(\theta), s_2(\theta)\right) = \left\{ \begin{array}{llll} 20, & \text{if} & s_1(\theta)=2 \ \ \text{or} \ \ s_2(\theta)=2 & \text{(e.g. flow on route 2 is 1)} \\ 40, & \text{if} & s_1(\theta)=2 \ \ \text{and} \ \ s_2(\theta)=2 & \text{(e.g. flow on route 2 is 2)} \end{array} \right\} \tag{4.6}$$

We assume that traveler i aims to maximize his/her individual travel utility (payoff) given by the following expression:

$$J_i\left(s_1(\theta), s_2(\theta)\right) = \left\{ \begin{array}{ll} \overline{U} - \alpha_i \tau_1\left(s_1(\theta), s_2(\theta)\right) - \theta, & \text{if } s_i(\theta)=1, \text{ route 1 (tolled);} \\ \overline{U} - \alpha_i \tau_2\left(s_1(\theta), s_2(\theta)\right), & \text{if } s_i(\theta)=2, \text{ route 2 (untolled);} \\ 0, & \text{if } s_i(\theta)=3, \text{ route 3 (do not travel)} \end{array} \right\} \tag{4.7}$$

In Equation (4.7), $\overline{U}$ represents the trip utility when making the trip to destination B, $\tau_p(\cdot)$ denotes the path p travel time depending on the chosen strategies by all travelers, $p = \{1, 2, 3\}$, and α_i represents the *class-specific value of time* of traveler i. Note that negative net utilities on route 1 and 2 imply that one will not travel, i.e. if the cost (disutility) of making the trip is larger than the utility of the trip itself the traveler will decide not to make a trip. It is necessary to choose the value for the trip utility $\overline{U}$ in a way to capture the choice of travelers not to make a trip. According to Equation (4.7), the value of trip utility should be smaller than the multiplication of α_i and $\tau\left(s_1(\theta), s_2(\theta)\right)$.

Since we only consider *pure strategies*, we do not have to know the shape of the travel time functions on the whole range (only two points are needed). In our examples we thus consider discrete flows instead of continuous flows, hence Wardrop's first principle, according to which all travel utilities are equal for all used alternatives, may no longer hold

Table 4.1: Utility payoff table for travelers

		Strategy of traveler 2		
	(J_1, J_2)	$s_2 = 1$	$s_2 = 2$	$s_2 = 3$
Strategy of traveler 1	$s_1 = 1$	$(102 - \theta, 102 - \theta)$	$(150 - \theta, 90)$	$(150 - \theta, 0)$
	$s_1 = 2$	$(90, 150 - \theta)$	$(-30, -30)$	$(90, 0)$
	$s_1 = 3$	$(0, 150 - \theta)$	$(0, 90)$	$(0, 0)$

in this case. In fact, the more general equilibrium rule applies in which the minimum trip utility over all travelers is maximized (the problem of minimizing the maximum latency of flows in static networks with congestion. For a more detailed explanation see e.g. Correa et al. (2004).

In order to solve the optimal toll design game it is necessary to formulate the payoff table for the travelers. The utility payoff table has a bi-matrix formulation where the strategies of two players (player 1 and player 2) are presented (see Definition 7). Both travelers can play different strategies, in our case to choose different travel routes (route 1 - tolled, or route 2- untolled) or not to travel at all (route 3). The utility payoff values are computed for all strategy combinations of travelers. The utility payoff table, depending on the toll is given in Table 4.1 for both travelers, where the values between brackets are the payoffs for travelers 1 and 2, respectively (see the bi-matrix notation introduced in the previous section).

The explanation of the table is as follows. Let us consider the case where the traveler 1 chooses strategy $s_1 = 1$ and the traveler 2 chooses strategy $s_2 = 1$. Suppose that there is only one group of travelers with the value of time $\alpha = 6$. The value for the trip utility should be smaller than $\alpha \cdot \tau\left(s_1(\theta), s_2(\theta)\right)$. According to Equations (4.5) and (4.6) the net utility should be smaller than $\overline{U} = 240$. In this experiments we will use $\overline{U} = 210$. Then the payoff for the traveler 1 will be, according to the Equation (4.7), $210 - 6 * 18 - \theta = 102 - \theta$, as well as for the traveler 2 (see Table 4.1). In this way, all other payoff values are determined for all strategy combinations and for both travelers.

After that the utility payoff table for the travelers is established, it is necessary to formulate the payoff function for the road authority. Different policy objectives of the road authority can be applied depend on what the road authority would like to achieve. Our experiments are organized in the following way. In the following sections, in experiments we will consider three different road authority's objectives: total travel utility, generating revenues and social surplus. A case-study with heterogeneous users is also presented.

4.3 Case Study 1: Policy objective of the road authority: Maximizing total travel utility

In this section the policy objective of the total travel revenues is considered. For this policy objective, the aim is to maximize total travel utility of the travelers. Three different

Table 4.2: Utility payoff for the road authority for total travel utility objective

		Strategy of traveler 2		
	$R(\theta)$	$s_2 = 1$	$s_2 = 2$	$s_2 = 3$
Strategy of traveler 1	$s_1 = 1$	$204 - 2\theta$	$240 - \theta$	$150 - \theta$
	$s_1 = 2$	$240 - \theta$	-60	90
	$s_1 = 3$	$150 - \theta$	90	0

game concepts are applied: monopoly ('social planner' game), Stackelberg and Cournot game, respectively. Stackelberg game is illustrated as a normal-form (matrix) game as well as an extensive-form (tree) game. For more information about normal and extensive form game see Chapter 3. The comparison among different game concepts is given at the end of this section.

Let us add the road authority as a player in the optimal toll design game. The policy objective of maximizing total travel utility is chosen because in this experiments we consider trip choice of travelers. That means that if we choose e.g. the policy objective of minimizing the total travel time on the whole network, then the optimal solution will be to set (very) high tolls on the network. In that case, travelers will certainly choose not to travel due to high travel costs. It is clear that this policy objective will result in a not realistic solution. That was the reason that we assume that the road authority aims at maximizing the total travel utility of the travelers, i.e.

$$\overline{R}\left(s^*(\theta), \theta\right) = J_1\left(s^*(\theta)\right) + J_2\left(s^*(\theta)\right) \tag{4.8}$$

According to Equation (4.8) the payoff values of the road authority are determined. Let us suppose that the value of time of travelers is same for both travelers, that is $\alpha_i = 6$. The payoffs of the road authority (see Table 4.2) depend on the strategy that the road authority applies, as well as on the strategies the travelers choose. For example, if traveler 1 choose strategy $s_1 = 1$ (tolled route) and traveler 2 choose the same strategy $s_2 = 1$ (tolled route) then the payoff for the road authority will be (according to Equation (4.8) and Table 4.1) $(102-\theta)+(102-\theta) = 204-2\theta$. The payoff values of the road authority if the objective is total travel utility for all other strategy combinations of travelers is determined and presented in Table 4.2.

Let us use these utility payoff tables (for travelers, Table 4.1 and for the road authority, Table 4.2) to solve the previously defined game concepts (see Chapter 3): monopoly game, Stackelberg game and Cournot game, respectively.

4.3.1 Monopoly (social planner) game

In the monopoly game, as it was stated before (see Section 3.8), the road authority sets the tolls as well as the travel decisions of the travelers such that his payoff is maximized given his objective (in this case total travel utility). Since the travel utility always decreases with

Table 4.3: Utility payoff for the road authority if toll=0

		Strategy of traveler 2		
	$R(\theta)$	$s_2 = 1$	$s_2 = 2$	$s_2 = 3$
Strategy of traveler 1	$s_1 = 1$	204	240	150
	$s_1 = 2$	240	−60	90
	$s_1 = 3$	150	90	0

increasing θ, the optimal strategy for the road authority should be $\theta^* = 0$. The utility payoff table for the road authority if $\theta = 0$ is presented in Table 4.3. In this case, the maximum utility can be obtained if the travelers distribute themselves between routes 1 and 2, leading to the two optimal strategies: $s^* = (1, 2)$ and $s^* = (2, 1)$. Hence, in this system optimum solution, the total travel utility achieved in the system is 240. Note that this optimum would not occur if travelers had free choice, since $\theta = 0$ yields a Nash-Wardrop equilibrium for both travelers choosing route 1, leading to a total utility of 204. Therefore, some pricing mechanism will be necessary to influence travelers to change their travel strategies (in this case one of them has to choose route 2).

4.3.2 Stackelberg game solution

In this game the travelers will maximize their own travel utility, dependent on the toll set by the road manager. First, the payoff table for the travelers will be determined (according to Table 4.1) for all values of tolls θ. In this framework we suppose that tolls cannot be negative. The strategy set of the road manager is assumed to be $\Theta = \{\theta |\ \theta \geq 0\}$. In this example, we will illustrate tolls within the following range: $0 \leq \theta \leq 170$.

Payoff table for travelers

An illustration of the utility payoff table if $\theta = 12$ is given in Table 4.4. The Nash solution of the optimal toll design game if $\theta = 12$ is as follows. First, let us consider traveler 1. If the traveler 2 will choose strategy $s_2 = 1$ (that is the tolled route) then the traveler 1 can choose among three available strategies, that is $s_1 = 1$, $s_1 = 2$, or $s_1 = 3$, respectively. These possible strategies for traveler 1 yield at corresponding utility payoff values, that is, $J(s_1 = 1, s_2 = 1) = 90$, $J(s_1 = 2, s_2 = 1) = 90$ and $J(s_1 = 3, s_2 = 1) = 0$. Because the traveler 1 would like to maximize his own utility, clearly he will be motivated to choose $s_1 = 1$, resulting at the payoff value $J(1, 1) = 90$ and/or $s_1 = 2$, resulting at the same payoff value, $J(1, 2) = 90$. For an illustration, these chosen strategies are printed bold in Table 4.4. The same procedure is performed for the traveler 2 if the strategies for traveler 1 are fixed. The results are presented in Table 4.4. Multiple Nash equilibrium solutions are denoted with a star (*). When applying the definition of dominant strategies (expressions (4.3) and (4.4)), the strategy $s(12) = (1, 1)$ is eliminated while the optimal strategies for travelers, $s^*(12) = (1, 2)$ and $s^*(12) = (2, 1)$ are chosen.

Payoff for the road authority

Table 4.4: Utility payoff table for travelers if toll=12

		Strategy of traveler 2		
	(J_1, J_2)	$s_2 = 1$	$s_2 = 2$	$s_2 = 3$
Strategy of traveler 1	$s_1 = 1$	**90**, **90***	**138**, **90***	**138**, 0
	$s_1 = 2$	**90**, **138***	−30, −30	90, 0
	$s_1 = 3$	0, **138**	0, 90	0, 0

Table 4.5: Utility payoff for the road authority if toll=12

		Strategy of traveler 2		
	(J_1, J_2)	$s_2 = 1$	$s_2 = 2$	$s_3 = 3$
Strategy of traveler 1	$s_1 = 1$	180	228*	138
	$s_1 = 2$	228*	−60	90
	$s_1 = 3$	138	90	0

After the Nash solutions have been determined for the travelers, the payoff table for the road authority (according to Table 4.2) will be computed for different toll values (within given boundaries). An illustration for the toll value $\theta = 12$ is given in Table 4.5. According to the table, the optimal value of the policy objective of maximizing total travel utilities that the road authority can achieve if the toll is $\theta = 12$ is 228.

In the same way the computation for other toll values $0 \leq \theta \leq 170$ is performed and results are presented in Figure 4.2. Figure 4.2 illustrates the total travel utility $\overline{R}\left(s^*(\theta), \theta\right)$ for different values of θ with the corresponding optimal strategies $s^*(\theta)$ chosen by the travelers.

From Figure 4.2 we can derive some conclusions about optimal toll strategies for travelers and for the road authority. For $0 \leq \theta < 12$ both travelers choose route 1 (because this strategy combination will each give maximal benefits). According to Figure 4.2, if $12 \leq \theta \leq 150$, travelers distribute themselves between route 1 and 2, while for $\theta \geq 150$ one traveler will take route 2 while the other traveler will not travel at all. Clearly, the optimum for the road manager is $\theta^* = 12$ yielding a total travel utility of 228.

Note that the objective function $\overline{R}\left(s^*(\theta), \theta\right)$ has a discontinuous shape (at toll value θ =12). The reason is that pure strategies of players are used in this experiment. In pure strategies each player is assumed to adopt only a single strategy. The consequence of using pure strategies (instead of mixed strategies) is existence of *multiple Nash equilibrium* solutions for one strategy combination. In that case the principle of *dominant strategies* is applied to determine the optimal solution of the game.

Two-stage Stackelberg optimal toll design game

A two-stage, 'leader-follower' Stackelberg optimal toll design game comprises two main steps:1) formulating the optimal toll design game (defining players, objectives of the players, and payoff values for players) as an extensive-form (tree) game and 2) solving the

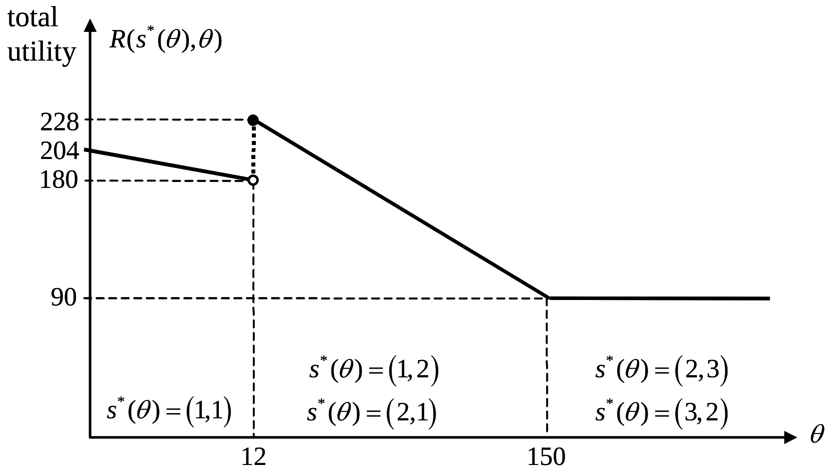

Figure 4.2: Total travel utilities depending on toll value

optimal toll design game as an extensive-form game (determining the optimal strategies for all players, i.e. the road authority and the travelers resulting in the optimal solution of the optimal toll design game for all players). Therefore, the optimal toll design game can be explain illustrating these two parts.

For representation of the two stage optimal toll design game the *extensive form* of the game (instead of normal form, see Table 4.4) will be used because the extensive form (Figure 4.3) is more appropriate for dynamic or (quasi)dynamic games where there exists a sequence in playing strategies between players. For static games or one-stage games a normal form of games is more appropriate.

Formulation of the two-stage optimal toll design game As stated earlier, there are $N+1$ players in the optimal toll design game (namely, many followers and one leader) with different interests. On the upper level of the road pricing problem, the road authority ('leader') may choose different strategies for the toll levels on tolled route 1. Let us suppose that a range of strategies of the road authority, namely $\theta \geq 0$ are considered (as indicated in Figure 4.3). Travelers constitute the lower level of the road pricing problem ('followers'), where payoffs of all the strategy combinations (included tolls from the upper level) will be expressed according to Table 4.1 in the following way.

Let us consider the case where the road authority sets toll $\theta = 5$ (see Figure 4.3). After the toll is being set by the road authority, the travelers react on this toll by choosing routes. The payoff values for every strategy combination of the travelers are determined using Equation (4.7) (e.g. if the road authority sets toll $\theta = 5$, then the travelers can choose their strategies (route 1, route 1) and the payoff values are (97, 97) for both travelers). As stated before, Wardrop's first principle can also be restated as to maximize the minimal individual utility, the payoff for the route combination will be the minimum of the both individual trip utilities (see Equation (4.9)).

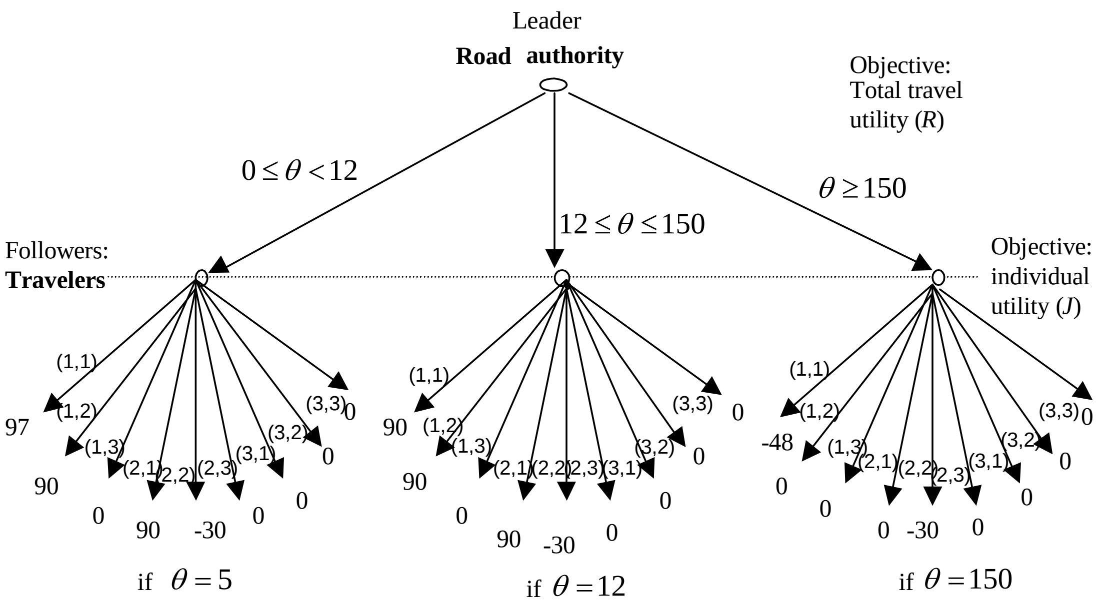

Figure 4.3: Formulation of the two-stage optimal toll design game

$$\overline{J}(s_1, s_2) = \min\{J_1(s_1, s_2), J_2(s_1, s_2)\} \tag{4.9}$$

In that way the minimum between these two value (97, 97) will be chosen. Because of simplicity, we will indicate only the chosen (minimal) value at Figure 4.3. The same procedure is done for all other travelers' strategies and minimal payoff values are presented in Figure 4.3. As an illustration, cases where $\theta = 12$ and $\theta = 150$ are given.

Each arrow in Figure 4.3 represents a possible strategy combination for travelers including tolls from the upper level. The formulation of the optimal toll design game (with different players, objectives, strategies, and payoff values for travelers) is shown in Figure 4.3. After that the optimal toll design game is formulated it is necessary to solve the game and determine the optimal strategies for both players as well as their payoffs.

Solving the Stackelberg optimal toll design game In the first stage the road authority as a leader chooses the toll strategy from the range of possible toll strategies.

Let us consider the case where the road authority chooses the toll option $\theta = 5$. If $\theta = 5$, then the travelers will choose their optimal travel strategy among all available strategies. From all possible travel options (strategy combinations) for the travelers (taking into account toll value resulting from the first stage of the game) the best options for the travelers will be chosen. In our example in Figure 4.4 the best strategy combination is (1, 1) yields the payoff values of (97, 97). According to Wardrop's first principle (described in the previous section) the minimum value will be chosen, that is 97. Because of simplicity we indicated only the minimal value (97) in Figure 4.4. Taking this outcome from the lower level into account, the payoff value for the road authority is determined ($R = 194$).

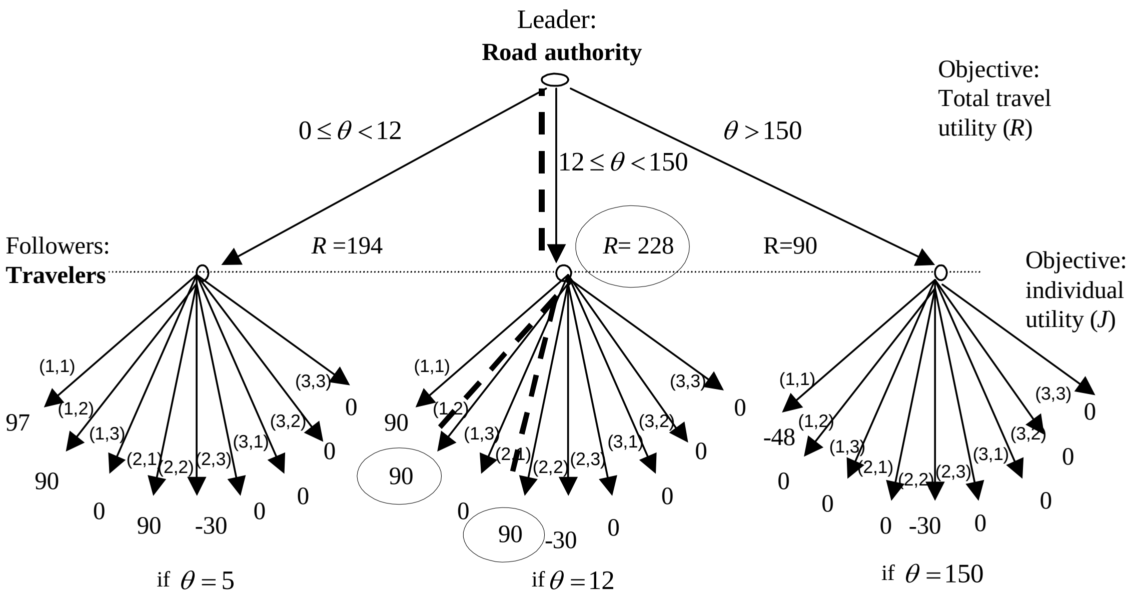

Figure 4.4: Solution of the two-stage optimal toll design game

The same procedure is performed for all others (toll) strategies of the road authority. From Figure 4.4 it can be seen that if $\theta = 12$ (Nash equilibrium solution), the minimum payoff for the travelers is 90 if they choose to travel on route 1 and route 2, while the maximum payoff for the road authority is 228 (dashed lines). In this way the optimal toll value which maximize the objective of the road authority is determined. The solution of the game is indicated with dotted lines while payoff values for the travelers and the road authority are indicated by circles.

In this section we show how to determine an optimal solution using Stackelberg game concept in a game theory (extensive-form game) framework.

4.3.3 Cournot game

Solving the optimal toll design problem as a Cournot game requires a *joint utility payoff table* for the road authority as well as travelers. The joint utility payoff table should have all strategy combinations for all players, because they will choose their strategies simultaneously.

The joint payoff table is formulated as follows. The utility payoff values for the travelers combined will be the illustrated by a minimum value of the individual trip utilities (first part of the expression in the payoff matrix in Table 4.6). The second part in the expression in Table 4.6 is the payoff value for the road authority.

As before, the utility payoff for the road authority is the total travel utility. The formulation of the Cournot game is shown in Table 4.6 for a few values of θ (while all values $\theta \geq 0$ are possible strategies).

Table 4.6: Cournot solutions of the optimal toll design game

		Strategy of the road authority		
		$\theta = 0$	$\theta = 15$	$\theta = 170$
Combined strategies for both travelers	$s = (1, 1)$	$(\mathbf{102}, \mathbf{204})^*$	$(87, 174)$	$(-68, -136)$
	$s = (1, 2)$	$(90, \mathbf{240})$	$(\mathbf{90}, 225)$	$(-20, 70)$
	$s = (1, 3)$	$(0, \mathbf{150})$	$(0, 135)$	$(-20, -20)$
	$s = (2, 1)$	$(90, \mathbf{240})$	$(\mathbf{90}, 225)$	$(-20, 70)$
	$s = (2, 2)$	$(-30, \mathbf{-60})$	$(-30, \mathbf{-60})$	$(-30, \mathbf{-60})$
	$s = (2, 3)$	$(0, \mathbf{90})$	$(0, \mathbf{90})$	$(\mathbf{0}, \mathbf{90})^*$
	$s = (3, 1)$	$(0, \mathbf{150})$	$(0, 135)$	$(-20, -20)$
	$s = (3, 2)$	$(0, \mathbf{90})$	$(0, \mathbf{90})$	$(\mathbf{0}, \mathbf{90})^*$
	$s = (3, 3)$	$(0, \mathbf{0})$	$(0, \mathbf{0})$	$(\mathbf{0}, \mathbf{0})$

Table 4.7: Comparison of outcomes using different game concepts

	Optimal strategy		Maximum payoffs	
	Road authority	Travelers	Road authority	Travelers
Game Concepts	(θ^*)	$s^* = (s_1^*, s_2^*)$	$(\overline{R})$	(J_1, J_2)
Monopoly	0	(1, 2) (2, 1)	240	(90, 150) (150, 90)
Stackelberg	12	(1, 2) (2, 1)	228	(90, 138) (138, 90)
Cournot	0	(1, 1)	204	(102, 102)

After that the joint payoff table is formulated it is necessary to solve the table according to the Nash principle. First, the travelers will choose the maximal valued from all nine (minimal) payoff values for their possible strategies, if toll of the road authority is fixed. Then, the road authority will choose the maximum utility. The chosen utilities are highlighted in Table 4.6.

According to Table 4.6, multiple Nash equilibrium solutions exist (indicated with *). There is, however, one dominating strategy, being that the travelers both take route 1 and that the road manager sets zero tolls, yielding a total system utility of 204. As can be checked, the solution will still hold if we consider other θ values.

4.3.4 Comparison of games for the policy objective of maximizing the total time utility

Table 4.7 summarizes outcomes for different game concepts presented in the previous sections: monopoly, Stackelberg and Cournot game.

According to Table 4.7, the Stackelberg game yields a payoff for the road authority of 228, setting his strategy to $\theta = 12$. Corresponding payoff values for the travelers are $(90, 138)$ or $(138, 90)$ if they choose strategy $(1, 2)$ or $(2, 1)$, respectively. If the road

Table 4.8: Payoff table for the road manager for the objective of maximizing revenues

		Strategy of traveler 2		
		$s_2 = 1$	$s_2 = 2$	$s_2 = 3$
Strategy of traveler 1	$s_1 = 1$	2θ	θ	θ
	$s_1 = 2$	θ	0	0
	$s_1 = 3$	θ	0	0

authority has complete market power (i.e. monopoly game concept) then his payoff could be 240. If the road authority has equal market power compared to the travelers, only 204 can be reached.

Taking into account the nature of road pricing and results of the experiments conducted in the previous sections (Table 4.7) we conclude that the Stackelberg game is the most realistic game concept for the optimal toll design problem. That is the main reason that, in the next sections about policy objectives of the road authority, only the Stackelberg game is analyzed. It should be noted that practical studies (such as congestion charging in London, TfL (2004), and pricing in Singapore) are examples of the Stackelberg game in reality. More realistic game concept can be inverse Stackelberg game where tolls are function of the traffic flows in the network (and not constant of time-varying). For more information about inverse Stackelberg game see Stankova et al. (2006).

4.4 Case Study 2: Policy objective of the road authority: Maximizing total toll revenues

In this section the policy objective of maximizing total toll revenues on the network will be considered. The Stackelberg game concept (as the most realistic game concept) is applied to the optimal toll design problem.

Let us consider the road authority as a player, who tries to maximize revenues according to Equation (3.6). In this experiment, where we consider only two travelers and one tolled route, the policy objective of the road authority to maximize total toll revenues is as follows:

$$\overline{R}\left(s^*\left(\theta\right),\theta\right) = q_1\left(s^*\left(\theta\right)\right)\theta \tag{4.10}$$

where $q_i\left(s^*\left(\theta\right)\right)$ is the number of travelers using route 1 (tolled route) and θ is the toll imposed on that route.

Depending on the strategy that the road authority plays and depending on the strategies travelers play, the utility payoff table for the road authority is presented in Table 4.8.

Stackelberg game for the policy objective of maximizing total toll revenues will be applied to the optimal toll design problem. Figure 4.5 illustrates the revenues for different values of θ with the corresponding optimal strategies played by the travelers. When

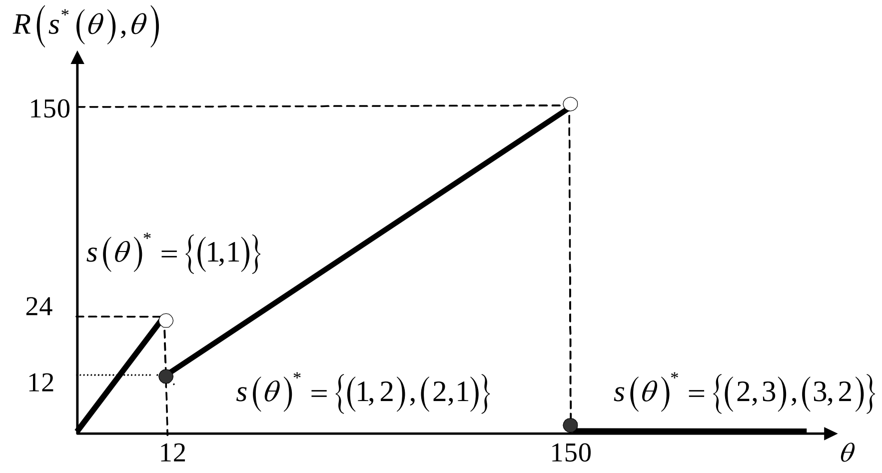

Figure 4.5: Payoff of the road authority depending on toll values for the policy objective of maximizing total toll revenues

$0 < \theta < 12$, travelers will both choose route 1. If tolls are higher, that is $12 < \theta < 150$, travelers distribute themselves between route 1 and 2, while for very high tolls, $\theta > 150$, one traveler will take route 2 and another traveler will not travel at all. Clearly, the optimum of the road authority is $\theta^* = 150$ yielding a total system utility of 240. If $\theta = 12$, then multiple Nash equilibriums exist, namely, travelers can choose strategy (1,1) but also strategies (1,2) and (2,1). In these cases we applied the principle of dominated strategies. The dominated strategies are indicated in Figure 4.5 with a full circle while non-dominated solutions are indicated by an empty circle. Similarly, if $\theta = 150$ travelers can choose between strategies (1,2) and (2,1) but also strategies (2,3) and (3,2).

4.5 Case Study 3: Policy objective of the road authority: Maximizing social surplus

In this section the policy objective of maximizing social surplus is presented and Stackelberg game concept is illustrated. This case study is the combination of the two previous described policy objectives of the road authority.

In this case, the road authority is assumed to maximize social surplus according to formula (3.8). For our special case with two travelers and only one route tolled, the formula is as follows:

$$\overline{R}(s^*(\theta), \theta) = J_1\left(s^*(\theta)\right) + J_2\left(s^*(\theta)\right) + q_1\left(s^*(\theta)\right)\theta \qquad (4.11)$$

Again, the strategy set of the road manager is assumed to be $\Theta = \{\theta | \ \theta \geq 0\}$. Utility payoffs for the road authority are presented in Table 4.9 depending on the strategy that the road authority chooses and depending on the strategies travelers play.

Table 4.9: Payoff table for the road authority for the social surplus objective

	$R(s(\theta),\theta)$	*Strategy of traveler 2* $s_2 = 1$	$s_2 = 2$	$s_2 = 3$
Strategy of traveler 1	$s_1 = 1$	204	240	150
	$s_1 = 2$	240	-60	90
	$s_1 = 3$	150	90	0

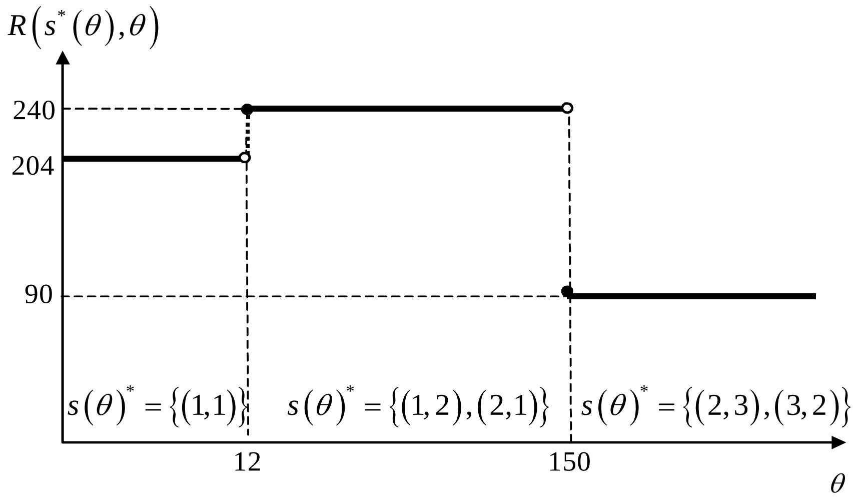

Figure 4.6: Utility payoff of the road authority depending on the toll values for the policy objective of maximizing social surplus

In this case, because the Stackelberg game is the most realistic game for the optimal toll design problem, we illustrate only the Stackleberg game for the policy objective of maximizing social surplus.

The Stackelberg game concept for the social surplus policy objective is as follows. Figure 4.6 illustrates the policy objective of maximizing social surplus for different values of θ with the corresponding optimal strategies played by the travelers. When $0 < \theta < 12$ travelers will both choose route 1 yielding a total social surplus of 204. If $12 < \theta < 150$ travelers distribute themselves between route 1 and 2, while for $\theta > 150$ one traveler will take route 2 and another traveler will not travel at all. Clearly, the optimum for the road authority yields a total system utility of 240 for any toll valued between 12 and 150. As in the previous subsection, multiple Nash solutions exist. Namely, if $\theta = 12$, travelers can choose strategy (1,1) but also strategies (1,2) or (2,1). The same happens for the toll value $\theta = 150$.

Table 4.10: Comparison of different policy objectives

	Optimal strategy		Maximum payoffs	
	Road authority	Travelers	Road authority	Travelers
Policy objective:	(θ^*)	$s^* = \left(s_1^*, s_2^*\right)$	$\left(\overline{R}\right)$	(J_1, J_2)
1. total travel utility	12	(1, 2) (2, 1)	228	(90, 138) (138, 90)
2. total toll revenues	150	(2, 3) (3, 2)	150	(90, 0) (0, 90)
3. social surplus	{12,150}	(1, 2) (2, 1)	240	({138, 0}, 90) (90, {138, 0})

4.6 Comparison among different policy objectives with regard to Stackelberg game

In this section the comparison between different policy objectives of the road authority is given with regard to the Stackelberg game concept (see Table 4.10).

Considering all three case studies (Sections 4.3, 4.4 and 4.5) with different policy objectives, it is shown that there exist different policy objectives that all can be applied depending on what the road authority would like to achieve. We apply only a subset of possible policy objectives of the road authority (maximization of total travel utility, maximization of total toll revenue and social welfare). Different policy objectives lead to different outcomes, both in terms of optimal toll to be set by the road authority as well as in utility payoffs for players. In MC-ICAM (2004), the toll revenue and social welfare objectives are analyzed using three different solutions (centralized, Non-cooperative and Stackelberg). The results show that Stackelberg equilibria achieved most of the improvements compared to the other game concepts.

Further, some conclusions can be driven:

- In the small number cases (as in examples used in this chapter), the objective function may have a non-continuous shape (jumps) which can be explained by using pure strategies in the model; in the case of large numbers, the objective function has a continuous shape (see Chapter 7);

- Applying pure strategies is the reason of existence of multiple optimal solutions (multiple Nash equilibria). In that case, a principle of dominated strategy can be applied to choose the optimal toll strategy.

Table 4.11: Payoff table for combined travelers

	$J = (J_1, J_2)$	*Strategy of traveler with high value of time* $s_2 = 1$	$s_2 = 2$	$s_2 = 3$
Strategy of traveler with low value of time	$s_1 = 1$	$(102 - \theta, 30 - \theta)$	$(150 - \theta, 10)$	$(150 - \theta, 10)$
	$s_1 = 2$	$(90, 110 - \theta)$	$(-30, -190)$	$(90, 0)$
	$s_1 = 3$	$(0, 110 - \theta)$	$(0, 10)$	$(0, 0)$

Table 4.12: Payoff table for the road manager

	R	*Strategy of traveler with high value of time* $s_2 = 1$	$s_2 = 2$	$s_2 = 3$
Strategy of traveler with low value of time	$s_1 = 1$	$132 - 2\theta$	$160 - \theta$	$150 - \theta$
	$s_1 = 2$	$200 - \theta$	-60	90
	$s_1 = 3$	$110 - \theta$	10	0

4.7 Case Study 4: Optimal toll design game with heterogeneous users

In previous case studies we assumed that both travelers will behave on the same way. In this case study, the heterogeneity of users in the optimal toll design game will be taken into account.

An illustration of heterogeneous users will be given in the following example. As a policy objective of the road authority the maximization of total travel utility can be chosen. Let us assume that travelers are different and that they have different value of time (α). Suppose that there are two different groups of travelers, one group with low value of time and the other with high value of time. The value of time for traveler 1 is $\alpha_1 = 6$ while the value of time for traveler 2 is higher, $\alpha_2 = 10$. The difference in values of time of travelers will play a significant role in the travel behavior (in this case, route and trip choice) of travelers. On the one hand we expect that the traveler with high value of time can afford to pay a higher toll in order to save his/her time. On the other hand, the traveler with low value of time should choose longer route in order to avoid to pay a toll.

The utility payoff table for both travelers is expressed according to Equation (4.7) and given in Table 4.11, where the values between brackets are utility payoffs for travelers 1 and 2, respectively. Please note that different values of time influenced the utility payoffs of different travelers.

The payoffs of the road authority are presented in Table 4.12. Because of different value of times for travelers, the payoff table of the road authority is not symmetric.

Monopoly game

In this case, the maximum utility can be obtained if (low value of time) traveler 1 takes route 2 and (high value of time) traveler 2 takes the tolled route 1, i.e. $s^* = (2, 1)$. Toll

Table 4.13: Payoff table for the road autority for the system optimum solution

	R	*Strategy of traveler with high value of time* $s_2 = 1$	$s_2 = 2$	$s_2 = 3$
Strategy of traveler with low value of time	$s_1 = 1$	132	160	150
	$s_1 = 2$	200	−60	90
	$s_1 = 3$	110	10	0

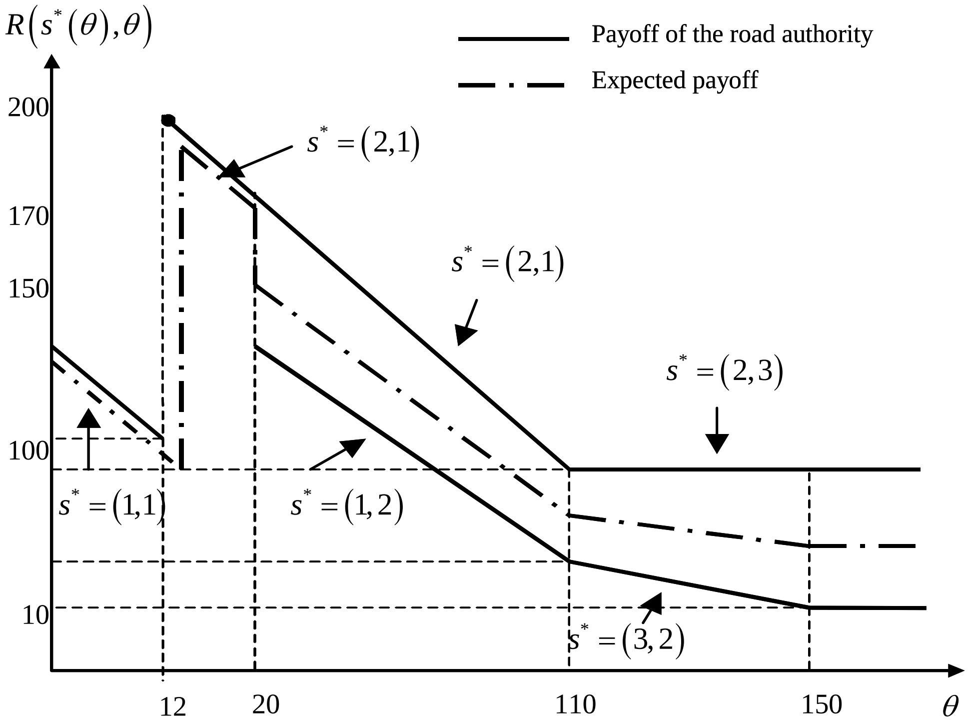

Figure 4.7: Total travel utilities depending on the toll value

value is set to zero. The utility payoff table for the road authority for the system optimum solution is shown in Table 4.13. Hence, in this system optimum, the total travel utility in the system is 200 and the optimal toll strategy for the road authority is $\theta^* = 0$.

Stackelberg game

Figure 4.7 illustrates the total travel utility for different values of tolls (θ) with the corresponding optimal strategies chosen by the travelers (s^*). The figure results from the utility payoff Table 4.12 determined for different values of θ ($\theta \geq 0$) for the traveler with high value of time and for the traveler with low value of time together.

For toll values $0 \leq \theta < 12$, the chosen strategy for the travelers is that both travelers take route 1, i.e. $s^*(\theta) = (1, 1)$ while the utility payoff for the road authority decreases with increasing toll levels. If $12 \leq \theta \leq 15$, the optimal strategy for the travelers is $s^*(\theta) = (2, 1)$, meaning that low value of time traveler 1 changes his route while the high value of time traveler 2 still stays on route 1. That can be explained by the fact that the high value of time traveler can afford to pay for a shorter trip.

For toll levels, $\theta \geq 20$, multiple (non-dominating) strategies exist for travelers, leading to different payoff utilities for the road authority. The dashed line indicates the expected payoff for the road authority (assuming a uniform distribution). If $20 \leq \theta \leq 110$, the optimal strategies for travelers are $s^* (\theta) = (2, 1)$ and $s^* (\theta) = (1, 2)$.

According to Figure 4.7, the optimum for the road manager is to choose $\theta = 12$ yielding a total travel utility of 188. It is shown how different groups of travelers (players) will react differently (change their travel behavior) on travel conditions depending on their value of time.

4.8 Summary and Conclusions

In this chapter we applied a micro-foundations modeling approach to analyze the behavior of individual actors in the optimal toll design game (introduced in Chapter 3) that can be especially useful for policy makers. The purpose of this chapter is to gain more insight into the behavioral effects of the road-pricing problem (route and trip choice) using different game theory concepts as well as different toll designs for multiple user classes. This chapter has studied optimal toll design game using a model that incorporates travelers' heterogeneity, a simple road network, and utility maximizing travelers, all in a game theory framework.

The new results from this chapter are the following. We presented a few experiments carried out on the optimal toll design game and presented three different game types (monopoly, Stackelberg and Cournot) in order to elucidate the essentials of the game theoretic approach. These game types were applied to the policy objective of minimizing total travel time on the network and exemplified on a simplistic demand-supply network system. The Stackelberg game (as a most realistic game) is applied on other policy objectives of the road authority. This clearly revealed differences in design results in terms of toll levels and payoffs for involved actors, being the road authority and network users. Heterogenous users are also included in the experiment showing differences in the travel behavior depend on their value of time.

Considering the optimal toll design game and the experiments presented in this chapter some conclusions can be derived:

- Different user classes lead to different outcomes of the games, both in terms of optimal toll as well as in payoffs for both the travelers and the road authority;
- Due to pure strategies, there may exist multiple optimal strategies (multiple Nash equilibria solutions);

- The objective function of the road authority will in general not have a nice shape, i.e. typically will be non-convex and non-continuous (in the pure strategy case).

The same analysis can be applied for different sorts of transport problems, e.g. to analyze two trams competing for the same track, two ships waiting for the dock, or two airplanes competing for their access for landing or taking-off. We can also establish some intuitive results for the optimal toll design game with heterogenous users:

- The more market power the road authority has, the higher is the payoff for the road authority;
- High value of time travelers will use the tolled route in more instances than low value of time travelers, and also accept a higher toll.
- It is shown how different groups of travelers (players) will react differently (change their travel behavior) on travel conditions depending on their value of time.

Similar results are found in Ubbels (2006) where travelers with high VOT accept pricing and are less price sensitive than the travelers with lower VOT.

Several limitations of the presented analysis may be relaxed in future work. The main idea of this chapter was to investigate road pricing as simple as possible (thus on a simplified network, with a small number of players and with simplified travel functions). In this case, the inner level game, describing the network equilibrium, and a game where individuals have market power is less relevant. However, for the situation where many individuals are included, on a bigger transportation network the formulation of the inner and outer games formulations make more sense.

The theory presented here can be extended to include other relevant travel choices such as e.g. departure time choice as well as imperfect information on the part of the road users. Mixed strategies can be applied instead of pure strategies. Different tolling schemes can be analyzed (e.g. step tolling, time-dependent tolls, anonymous tolling, discriminatory, etc.) and results should be compared in order to determine the optimal toll design pattern. Although a micro-economic perspectives can be used for better understanding the behavior of the individual actors in the road-pricing problem, for more practical use the presented game-theoretic analysis need to be translated into a modeling system with which tolling designs for more complex road networks can be computed. Tolls can be set as a function of the traffic flows in the network and the dynamic optimal toll design problem can be treated as an inverse Stackelberg game (Stankova et al. (2006)).

For that purpose, a bi-level optimization method will be used (see Chapter 5). The optimal toll design game is formulated as a two-stage Stackelberg game. So-called bi-level programming problem can be used to mathematically formulate the road pricing problem, where on the upper level is the road authority and on the lower level travelers. The whole problem can be formulated using a mathematical program with equilibrium constraints (MPEC) (see Chapter 5).

Part III

Macro-foundations of road pricing-
bi-level modeling framework

Chapter 5

Mathematical formulation of the dynamic optimal toll design (DOTD) problem

5.1 Introduction

While previous Chapters 3 and 4 considered a microscopic approach to solve the dynamic optimal toll design (DOTD) problem (interactions between individual actors), in this chapter a macroscopic approach will be used in contrast. In a macroscopic approach we consider interactions among groups of homogeneous actors. In other words, while a microscopic approach helps us to better understand the responses of individual actors to pricing, for more realistic situations (e.g. a larger number of travelers) a more general framework is needed. For that purpose, an aggregate level *macroscopic approach* to formulate the DOTD problem is developed in this chapter. The purpose of this chapter is to formulate the optimal toll design problem in a dynamic modeling framework in order to solve it and determine the optimal tolls on a transportation network.

It should be noted that there exists a similarity between the DOTD problem defined in this chapter and the game theoretic approach presented in Chapters 3 and 4. The similarity of the DOTD problem and the Stackelberg game is stated in different studeis, e.g. Yang & Bell (1998), Viti et al. (2003). Namely, the *mathematical program with equilibrium constraints* (MPEC) formulation of the DOTD problem can be interpreted as a game where the optimization problem is the top-level player (leader) while equilibrium constraints result from a N-player Nash game (followers).

The DOTD problem is considered as a *network design problem* (NDP) where the design variables to be determined are toll values for time periods, links and user-classes. Hence a formulation of the DOTD problem in a dynamic modeling framework (as a two-level mathematical formulation) is needed. The upper part of this DOTD problem, the pricing part, and the lower part, behavior of travelers, are modeled as two separate (but interrelated) optimization problems.

Since the lower level of the design problem corresponds to a typical equilibrium problem the DOTD problem is formulated as a dynamic MPEC, which is a special case of the bi-level programming problem. More about dynamic traffic equilibrium problems see in Bliemer (2001). Moreover, it is necessary to formulate policy objectives of the road authority and functional constraints on tolls.

This chapter will address the following questions. How to reformulate the game theoretic approach to include more realistic cases (e.g. a large number of travelers)? How can the DOTD problem be expressed as a dynamic bi-level optimization problem? How to define the design variables? How to mathematically express design variables and policy objectives of the road authority in the DOTD problem? What are the constraints on tolls in the DOTD problem? Does the DOTD problem formulation correspond to the MPEC formulation? The answers to these questions will be given in this chapter in order to formulate the DOTD problem in a dynamic modeling framework.

This chapter is organized as follows. First, the DOTD problem as a bi-level network design problem is outlined in Section 5.2. Then, the dynamic framework of the DOTD problem is described in Section 5.3. A mathematical formulation of the DOTD problem as a bi-level problem (BLP) is also presented. Furthermore, an MPEC formulation of the DOTD problem (where the lower level of the DOTD problem is an equilibrium problem) is given in this section. Policy objective functions of the road authority as well as constraints on tolls are presented in Section 5.5. In Section 5.4 toll constraints of the DOTD problem are derived. Finally, this chapter finishes with a summary and conclusion part (Section 5.6).

5.2 DOTD problem as a bi-level network design problem

The objective of a network design problem (NDP) is to determine properties of links (in our case link tolls) and nodes so that a given network performance measure (e.g. to minimize the total travel time on the given network) is optimized, while taking into account the travel behavior of the network users. Mathematically speaking, a network design problem is an optimization problem in which the aim is to determine the values of a set of design variables that will lead a system to an optimal state, according to a given objective.

Due to the increasing complexity of the model formulation, the NDP formulation has been recognized as one of the most difficult (yet challenging) problems in transport. For more information about NDP formulations we refer to e.g. Gumus & Floudas (2001), Meng et al. (2001). Network design problems have been proposed in various transport studies (e.g. optimizing the traffic network using a traffic control (green lights) or providing the travelers with information about network conditions). Our focus is to formulate the dynamic multi-user optimal toll design problem as an NDP.

In this case, the design variables of the DOTD problem to be determined are toll levels for time periods, links and user-classes on a transportation network. The purpose is to deter-

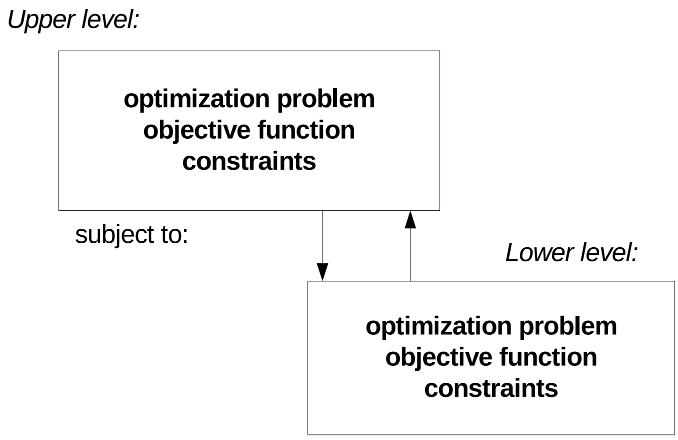

Figure 5.1: An illustration of a bi-level program (BLP)

mine optimal value(s) of the design variables that optimize a given policy objective of the road authority taking behavioral responses of travelers on the travel costs (including tolls) into account. The nature of the DOTD problem shows that it is a typical bi-level problem (BLP). The graphical illustration of a bi-level problem (BLP) is given in Figure 5.1. In general, a BLP is an optimization problem which is constrained by another optimization problem. Clearly, bi-level problems are a subset of multi-level optimization problems where the optimization considers only two (mutually interrelated) levels. Typically this kind of mathematical models arise when independent decision makers have *conflicting objectives*.

In recent years, bi-level problems gained a lot of attention in transport research studies. Bi-level optimization of transportation network problems can be found in Clegg & Smith (2001), Clegg et al. (2001), Cohen et al. (2002), Labbe et al. (1988). For a recent review, see Yang & Bell (2001). It is well known that transportation network optimization problems with user equilibrium can be well described as BLP (Shimizu et al. (1997)). In the work of Patriksson & Rockafellar (2002) a mathematical model for bi-level traffic management is presented. While there are lot of examples of bi-level problems in the static framework, only a few studies consider a dynamic framework such as e.g. traffic control (Van Zuylen & Taale (2004)) and origin-destination estimation (Lindveld (2004)). Some pricing applications of the dynamic bi-level formulation have been described in the literature (Abou-Zeid (2003), Viti et al. (2003)).

In our research we will also adopt the BLP approach and formulate the DOTD problem as a bi-level network design problem. However, we will extend the DOTD problem formulation to include class-specific travelers, and apply it for different policy objectives and different tolling patterns.

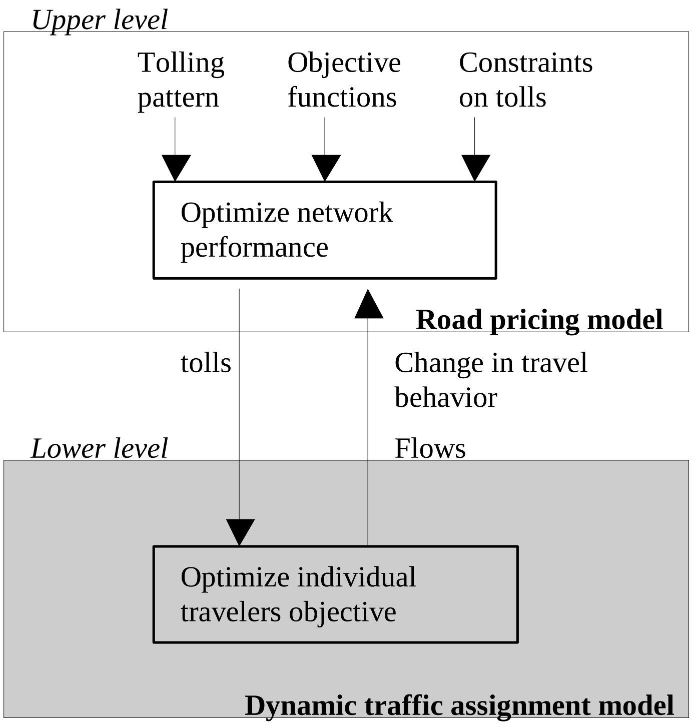

Figure 5.2: The bi-level framework of the dynamic optimal toll design problem

5.3 Framework of the DOTD problem formulation

In this section the framework of the DOTD problem formulation is outlined and explained. The perspective of the road authority as well as travelers is taken into account. However, the focus of this chapter will be on the road pricing model. The proposed framework consists of two different but interrelated models, namely: the road pricing model and the dynamic traffic assignment model (see Figure 5.2). The upper level of the DOTD problem concerns the optimization of the network performance according to the given policy objective of the road authority. At this level different policy objectives of the road authority can be considered and optimized depending on what the road authority would like to achieve (Chapter 2). To that end, different toll schemes can be applied in order to maximize the network performance (Chapter 2), depending on a set of constraints on the tolls. The connection between the upper and the lower level are the tolls (toll levels specified by time and space). Thus at the upper level the analyst seeks for the optimal toll values that optimize the authority's objective within given assumptions.

The lower level of the bi-level framework of the DOTD problem (Figure 5.2) concerns the travel behavior part of the DOTD problem. At this level, the modeling of the travelers' choice behavior is described including how travelers react on travel costs (including tolls) by changing their travel decisions. As can be seen from Figure 5.2 the optimization of the individual travelers' objectives takes place at this level. A bi-level model for toll optimization in static networks is studied by Brotcorne et al. (2001). In this thesis we focus on dynamic networks. More detailed explanation of the lower level of the DOTD problem is the subject of the next chapter of this thesis (Chapter 6).

5.3.1 MPEC problem - general formulation

According to various authors, the MPEC problem formulation is a special case of a bi-level programming problem where the upper level is an optimization problem while the lower level is an equilibrium problem. For more information about the MPEC formulation we refer to e.g. Luo et al. (1996), Lawphongpanich & Hearn (2004). The term equilibrium is adopted because the lower level of the MPEC of the DOTD problem typically models a certain equilibrium phenomenon (see Chapter 6). Thus an DOTD problem can be formulated as an MPEC problem. Mathematically speaking, the MPEC problem (expressions (5.1)-(5-3)) is formulated as a single level problem where the lower level constitutes equilibrium constraints of the MPEC formulation.

According to the literature study, the MPEC formulation has been applied in different studies. For example, the MPEC formulation for origin-destination matrix estimation can be found in Yang (1995), Maher et al. (2001), and Lindveld (2004). The theoretical study of the various MPEC formulations with different solution algorithms is presented in Luo et al. (1996). In Friesz et al. (2006), the dynamic optimal toll design problem for homogenous users is formulated using the MPEC approach. Our aim is to formulate the DOTD problem as an MPEC problem for different user-classes. For this purpose we first introduce the general mathematical formulation of an MPEC problem. $F(\cdot)$ is the objective function of the upper level, where u are the decision variables vector at the upper level while υ are the variables of the lower level.

$$\min_{u} F(u, \overline{\upsilon}(u)) \tag{5.1}$$

s.t.

$$G(u, \overline{\upsilon}(u)) \geq 0 \qquad \text{(set of constraints on the upper level variables)} \tag{5.2}$$

$$\Gamma(\overline{\upsilon}\,(u))(\upsilon - \overline{\upsilon}\,(u)) \geq 0, \quad \forall \upsilon \in \Psi(u) \qquad \text{(equilibrium constraints)} \tag{5.3}$$

where G is a multi-dimensional function which describes the set of constraints of the upper level, Γ is a function used to describe equilibrium constraints of the lower level, $\overline{\upsilon}\,(u)$ are optimal (lower level) values that hold at equilibrium and Ψ is defined as a set of all feasible values $\upsilon\,(u)$ for the lower level variables.

5.3.2 MPEC formulation of the DOTD problem

In this section the MPEC formulation of the stochastic, multi-class DOTD problem is given.

Let $\Omega = \{N, A\}$ denote a transportation network consisting of a set of nodes N and a set of directed links A. We denote the set of origin nodes with R, ($R \subseteq N$), and the set of destination nodes with S, $(S \subseteq N)$. Let (r, s) denote an origin-destination pair, $r \in R$, $s \in S$, while P^{rs} presents set of paths between origin nodes r and destination nodes s. An origin-destination pair can have one or more paths $p \in P^{rs}$ connecting the origin and the destination. Every path $p \in P^{rs}$ from r to s is comprised of one or more links. Let Y, ($Y \subseteq A$) denote set of links that can be tolled.

The DOTD problem can be formulated in continuous or discrete time (see more in Chapter 2). In this case, a discrete time formulation will be used, meaning that the studied time period T is divided into a certain number of small time intervals, $t \in T$. Departure time periods are denoted with $k \in T$. The whole formulation is class-specific, where m denotes a user class $(m \in M)$. The proposed notation is used in the remainder of this thesis.

Let $q \equiv \left[q_{pm}^{rs}(k)\right]$ be the vector of dynamic flow rates on all paths p departing from origins r to destinations s at all departure times k for traveler classes m, while $\theta \equiv [\theta_{am}(t)]$ is a vector of dynamic toll levels for all links a at time interval t for traveler classes m. The DOTD problem can be formulated as the following MPEC problem:

$$\min_{\Theta} F\left(\theta, \overline{q}(\theta)\right) \tag{5.4}$$

subject to:

$$\theta_{am}^{\min}(t) \leq \theta_{am}(t) \leq \theta_{am}^{\max}(t), \qquad \forall a \in Y\ ,\ \ \forall t \in T,\ m \in M \tag{5.5}$$

(upper level constraints)

$$\sum_{rs \in (RS)} \sum_{p \in P^{rs}} \sum_{k \in K} \sum_{m \in M} \overline{Q}_{pm}^{rs}(k, \theta) \cdot \left[q_{pm}^{rs}(k) - \overline{q}_{pm}^{rs}(k, \theta)\right] \geq 0, \qquad \forall q_{pm}^{rs}(k) \in \Psi \tag{5.6}$$

(lower level equilibrium constraints)

where $\overline{q}(\theta)$ is defined by the variational inequality (VI) problem (5.6), $\overline{Q} \equiv \left[\overline{Q}_{pm}^{rs}(k)\right]$ is a vector of *perceived equilibrium costs* on path p between origin r and destination s at departure time interval k for travelers classes m associated with equilibrium path flow rates $\overline{q}$, and Ψ is a set of feasible solutions of the equilibrium problem defined by network constraints of the lower level of the DOTD problem. The network constraints of the lower level of the DOTD problem as well as $\overline{Q}$ are explained more in Chapter 6. Minimal dynamic toll values $\theta_{am}^{\min}(t)$ and maximal dynamic toll values $\theta_{am}^{\max}(t)$ on link a at time interval t for traveler classes m are the lower and the upper boundaries on toll values in the DOTD problem, respectively.

In other words, the aim is to minimize the upper level objective function $F(\theta, \overline{q}(\theta))$ of the bi-level DOTD problem, subject to tolls constraints and user equilibrium constraints. It should be noted that in the lower level objective function, the aim is to determine the optimal path flow rates $\overline{q}$ as well as perceived equilibrium costs $\overline{Q}$ that satisfy the constraints of the lower level of the DOTD problem (Chapter 6).

While the objective function $F(\theta, \overline{q}(\theta))$ is formulated on the path level, the toll constraints of the DOTD problem are set on the link level. Thus, tolls in Equation (5.4) are path tolls, while tolls in constraints in Equation (6.5) are link tolls. Our motivation to use path-based formulations of policy objectives is because non-additive travel costs can be captured. Moreover, the travelers perceive the travel costs on a path level. The connection between these path-based and link-based tolls will be explained in Chapter 6 (Equation (6.6)) where we assume that the path tolls are additive. It should be noted that a more general formulation of the MPEC formulation of the DOTD problem with respect on path tolls can be easily derived. The DOTD problem formulated as an MPEC problem is a type of problem which is difficult to solve.

The general policy objective function $F(\theta, \overline{q}(\theta))$ of the bi-level formulation of the DOTD problem in Equation (5.4) can be different, such as minimizing total travel time in the network or maximizing total toll revenues (see more Chapter 2). These different policy objectives will be formulated in Section 5.4. The constraints of tolls, as presented in inequalities (5.5) will be examined in more detail in Section 5.4.

5.4 Toll constraints

Functional constraints of policy objectives of the road authority are already discussed in general in Chapter 2. However, in this section the mathematical formulation of the functional constraints is given. We state again the main constraint on tolls (5.5) while all other constraints are special cases of this constraint.

Lower and upper bounds on link toll values $\theta_{am}(t)$:

$$\theta_{am}^{\min}(t) \leq \theta_{am}(t) \leq \theta_{am}^{\max}(t), \qquad \forall a \in A, \ \ \forall t \in T, \ \forall m \in M \tag{5.7}$$

where $\theta_{am}^{\min}(t)$ is the lowest value allowed for the toll on link a when entering the link at time t for traveler class m, and where $\theta_{am}^{\max}(t)$ is the corresponding highest allowable toll.

The following specific constraints are all specific cases of constraint (5.7) by choosing appropriate levels for $\theta_{am}^{\min}(t)$ and $\theta_{am}^{\max}(t)$:

- nonnegativity of tolls, $\theta_{am}^{\min}(t) \geq 0, \quad \forall t \in T, \quad \forall a \in A\,,\ \forall m \in M$
- maximum toll levels, $\theta_{am}^{\max}(t) = \overline{\theta}_{am}^{\max}(t), \quad \forall t \in T, \quad \forall a \in A\,,\ \forall m \in M$
- only specific links are tolled, $\theta_{am}^{\min}(t) = \theta_{am}^{\max}(t) = 0, \quad \forall t \in T, \quad \forall a \notin Y, \forall m \in M$

  where a set of tolled links is denoted with Y, $Y \subset A$.
- only specific time periods are tolled, $\theta_{am}^{\min}(t) = \theta_{am}^{\max}(t) = 0, \quad \forall t \notin T', \forall m \in M$

  where a set of time periods which is tolled is denoted with T', $T' \subset T$.
- restrictions in time-varying tolls,

$$\begin{aligned} &\theta_{am}^{\min}(t) = \theta_{am}(t-1) - \Delta\,, \\ &\theta_{am}^{\max}(t) = \theta_{am}(t-1) + \Delta\,, \quad \forall t \in T, \quad \forall a \in Y\,,\ \forall m \in M \end{aligned} \tag{5.8}$$

  where Δ is the maximal value of toll that can increase or decrease in one time interval (e.g. 0.5 [eur]).

It should be mentioned that this framework can be easily extended for tolls formulated on the path level (suitable for e.g. parking fees, zone tolling, etc.). However, we will restrict ourselves to consider only link tolls in this research.

5.5 Policy objective functions

In this section, the focus is on the objective functions of the upper level of the DOTD problem, which reflect policy goals of the road authority.

Different policy objective functions of the road authority can be chosen, depending on what the road authority would like to achieve (e.g. to minimize the total travel time on the network, to maximize social benefit, or to maximize safety on the network, etc., see more in Chapter 2). A distinction between the true policy aim and the way of modeling policy is needed. Thus, if the true policy objective is to maximize revenues, then an elastic demand model is necessary, otherwise the revenues can be maximized by setting infinite tolls on all links in the model. In other words, one should offer travelers route alternatives that are untolled. Or, if the true policy objective is to minimize the total network cost, then a model with given fixed travel demand will be appropriate. For more information we refer to Yang & Lam (1996). In the case with elastic demand, this policy objective might be achieved through the minimization of total travel demand, and in that case the solution can reduce the travel demand to zero through an unrealistically high toll charge.

In this chapter the following policy objectives are mathematically formulated: minimization of total link travel time on the network and maximization of total toll revenues, respectively. The motivation to use these two objectives is to illustrate and give examples of 'conflicting' policy objectives in the DOTD problem leading to very different resulting toll levels. While in the case of revenue generation the aim is to attract as many travelers as possible, in the case of travel time minimization the aim is to spread the travel demand over time and links more evenly. Any other policy objective can merely replace the objective function, while the rest of the formulations remains the same.

Objective 1: Minimization of total travel time on the network

The aim of this policy objective is to achieve such a distribution of the total travel demand (path flows) over time and space such that the whole network performance is optimized, measured as the total travel time on the network. As a result, the optimal toll values which lead to the minimization of the total travel time should be determined. This policy objective is widely used in road pricing transportation studies (Teodorovic & Edara (2007), Yang & Zhang (2003), Cantarella et al. (2005)).

The mathematical formulation of the policy objective of the minimization of the total travel time on the network is given in Equation (5.9). This objective function is substituted with the general objective of the upper level of the bi-level problem given in (5.4).

$$F_1\left(\overline{q}\left(\theta\right)\right) = \sum_{(rs)\in RS}\sum_{p\in P^{rs}}\sum_{k\in K}\sum_{m\in M}\tau_{pm}^{rs}\left(k\right)\cdot\overline{q}_{pm}^{rs}\left(k,\theta\right) \tag{5.9}$$

The path-based objective function $F_1\left(\overline{q}\left(\theta\right)\right)$ is expressed as summed products of path travel times $\tau_{pm}^{rs}\left(k\right)$ and equilibrium path flow rates $\overline{q}_{pm}^{rs}\left(k,\theta\right)$, which depend on toll θ, summed over all paths, user-classes, and time periods.

The values for travel times and flows are determined by the lower level equilibrium problem, which is explained in Chapter 6.

Objective 2: Maximizing total toll revenues on the network

Another possible policy objective which the road authority may apply is the maximization of the total toll revenues on the network. We assume that travelers have untolled alternatives available, otherwise this objective would be achieved by setting infinite tolls in our inelastic demand model. In this case, on the one side, if no tolls are imposed on the network, no revenue will be generated. On the other side, if there are no travelers on tolled paths, then the generation of toll revenues is equal to zero. Clearly, there will exist *optimal* toll values which will maximize this policy objective of the road authority.

The mathematical formulation of the policy objective of maximizing revenues on the network is as follows:

$$F_2\left(\theta,\overline{q}\left(\theta\right)\right) = -\sum_{(rs)\in RS}\sum_{p\in P^{rs}}\sum_{k\in K}\sum_{m\in M}\theta_{pm}^{rs}\left(k\right)\cdot\overline{q}_{pm}^{rs}\left(k,\theta\right) \tag{5.10}$$

Clearly, in the case of maximization of the objective function (5.10), the sign of the function should be negative (conform minimization problem (5.4)). The objective function $F_2(\theta, q(\theta))$ has two different variables, namely toll levels θ and equilibrium path flow rates $\overline{q}$ on the network, in which the latter also (indirectly) depend on toll levels. The objective function is expressed as a multiplication of path toll values at time k, $\theta_{mp}^{rs}(k)$, and the number of travelers leaving at time k, $\overline{q}_{pm}^{rs}(k)$ over all origin-destination pairs $(rs) \in RS$, paths $p \in P^{rs}$, departure times $k \in K$, and for user classes $m \in M$.

5.6 Summary and Conclusions

In this chapter the dynamic optimal toll design problem is formulated as a bi-level problem with pricing design on the upper level and behavior of travelers on the lower level. The DOTD problem is classified as a bi-level network design problem. Since the lower level of the DOTD problem represents an equilibrium problem, it has been proposed to formulate the DOTD problem as an MPEC problem. The objective of the upper level is to optimize network performance by setting optimal tolls while the objective of the lower level is to optimize individual travelers' objective. Different policy objectives are formulated in this chapter: maximizing of total toll revenues and minimizing of total travel time on the network. The policy objectives are formulated subject to toll constraints.

In literature, the optimal toll design problem is formulated using the MPEC for homogenous users. However, we formulate the DOTD problem for different user-classes. It should be noted that we only formulate the optimal toll design problem in *dynamic* framework, but do not solve it using sophisticated algorithms. Thus another promising line of future research would be to develop solution methods to DOTD problem (see e.g. Ceylan & Bell (2005)).

In summary this study has revealed several important contributions. The DOTD problem is classified and formulated as a network design problem (NDP) and bi-level problem (BLP). Moreover, the nature of the lower level of the DOTD problem as an equilibrium problem led us to an MPEC formulation.

In the next chapter, the lower level of the DOTD problem will be explained in more detail. The behavioral part of the DOTD model captures how travelers react on travel costs (including tolls given from the upper level of the DOTD problem). In addition, the whole bi-level DOTD problem will be solved.

Chapter 6

Mathematical formulation of the travelers' behavior of the DOTD problem

6.1 Introduction

While in the previous chapter (Chapter 5) the pricing part of the dynamic optimal toll design (DOTD) problem is considered, the focus of this chapter is on the behavioral part. The behavioral part of the DOTD problem describes travel choices of users. As we noted in the previous chapter, the behavioral part of the DOTD problem is a typical equilibrium problem where different travelers' choices can be considered.

The DOTD problem is described as a dynamic traffic assignment (DTA) model consisting of a travel choice model and a dynamic network loading model. In this chapter the DTA model developed by Bliemer (2001) is taken as a base for further development for road pricing. Our focus is on the travel choice model and especially on needed extensions to capture road pricing. However, we consider not only route choice but also *departure time choice* of travelers, because it is known from the literature that due to (time-varying) tolls the main responses of travelers are departure time changes and route changes, respectively. For more information see e.g. Van Amelsfort et al. (2005b). More precisely, travel choices are assumed to be influenced by generalized travel costs, which among others consist of travel times, schedule delays, and toll costs. The behavioral part of the DOTD model will be formulated as a variational inequality (VI) problem leading to a *stochastic* dynamic user equilibrium. An overview of dynamic equilibrium network designs is given in Friesz & Shah (2001).

Comparing with the game theory approach (Chapters 3 and 4), in this approach we assume that the travel demand on the traffic network is *fixed* (hence the total travel demand is given). The motivation for such a choice is to consider mainly so-called *mandatory trips* (e.g. work or educational trips) instead of optional trips (e.g. leisure or social trips). With mandatory trips we consider trips where the travelers have more obligations to take these particular trips. In other words, the focus is on peak periods, in which mainly mandatory trips are made. According to Ubbels (2006) and MuConsult (2002), there are considerable differences between trip purposes, with commuting generally being least sensitive when the fare is time-independent. When policy makers aim to affect peak-time road traffic, a time differentiated measure seem to be most appropriate. According to Ubbels (2006), commuting trips are necessary and hard to replace because working at home or not making the trip are not serious options for most of the travelers.

The following questions are addressed in this chapter. How to modify the standard DTA model to capture (dynamic) road pricing? Which modifications of generalized travel cost functions are needed to include tolls? How to express heterogeneity of travelers?

The main contribution of this chapter is the modification of the generalized travel cost function to capture (time-varying) pricing. Namely, tolls from the pricing level of the DOTD problem (see Chapter 5) should be somehow captured in the generalized travel cost function. Besides this, heterogeneity of travelers is also taken into account.

The outline of this chapter is as follows. First, DTA model formulations are given in Section 6.2. After that, a modeling framework with extensions of the DTA model for road pricing is presented in Section 6.3. Furthermore, mathematical formulations of the DTA model to capture road pricing as well as the specification of the generalized travel cost function describing travelers' behavior (path and departure time choice) are given in Section 6.4. Additionally, a VI problem formulation with the focus on dynamic stochastic user equilibrium (DSUE) is stated in this section. The dynamic network loading (DNL) model is briefly presented in Section 6.5. Finally, the summary and conclusions are given in Section 6.6.

6.2 DTA problem formulations

A DTA model describes the interaction between the infrastructure supply and travel demand, based on choice behavior of the travelers. Analytical DTA models can be grouped into different categories depending of their problem formulations: mathematical programming, see Merchant & Nemhauser (1978), Carey (1986), Carey (1987), Carey (1992), Janson (1991), Wie et al. (1995) and Xu et al. (1999); optimal control theory, see Friesz et al. (1989), Ran & Shimazaki (1989), Wie (1989), Wie et al. (1990), Wie (1991), Ran et al. (1993); and variational inequality, see Friesz et al. (1993), Wie et al. (1995), Ran & Boyce (1996), Chen (1999), Chabini (2001), Bliemer & Bovy (2003).

In recent years, the variational inequality (VI) approach has gained increasing attention among researchers. Compared with mathematical programming and optimal control theory, variational inequalities provide a more attractive approach to formulate DTA problems. Namely, with the VI approach asymmetry of cost functions (e.g. for different vehicle types) can be handled. Bellei et al. (2002) developed a variational inequality model

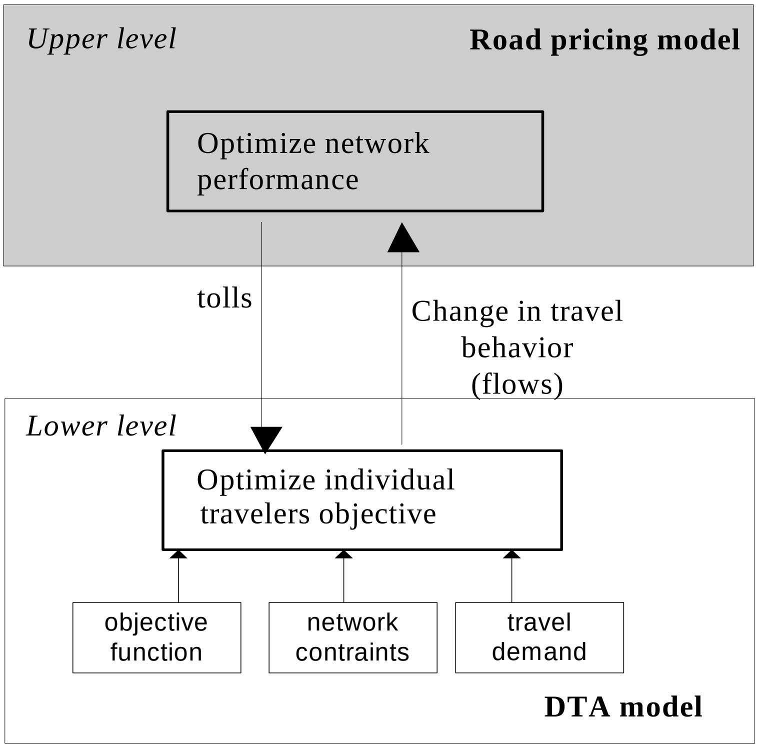

Figure 6.1: The bi-level framework of the DOTD problem with the focus on the DTA model

for network optimization in a multi-user and multimodal context; Liu & Boyce (2002) presented a variational inequality formulation of the marginal cost pricing problem for a general transportation network with multiple time periods. For a detailed review about VI formulations see Nagurney (1993) and Patriksson (1999).

In our research, we adopt the VI approach as the most promising approach for formulating and solving the DTA model for road pricing. More information about overviews of different formulations of the VI problem can be found in Bliemer (2001) and Boyce et al. (2001). For more information about DTA models in general we refer to nice overviews given in Peeta & Ziliaskopoulos (2001) and Szeto & Lo (2005).

6.3 Framework of the DTA model for road pricing

In this section we present the DTA framework which is the base for the extensions for road pricing. In Figure 6.1 the bi-level framework including the upper level of the DOTD problem (road pricing model) as well as the lower level (the DTA model) is illustrated. While in Chapter 5 the focus was on the upper level of the DOTD problem, now the lower level will be discussed in more detail.

As can be seen from Figure 6.1, the lower level of the DOTD problem represents travel behavior (route choice and departure time choice) where inputs to the model are the objective functions of individual travelers, the network constraints, and the travel demand. The aim is to determine which extensions and modifications are needed to capture road pricing. The main extension is the modification of the generalized travel cost function to capture different tolls from the upper level of the DOTD problem. In addition, the generalized travel cost function should express heterogeneity of travelers: e.g. differences in value of time (VOT) and value of schedule delay (VOSD). For an overview about VOT and VOSD see e.g. Lam & Small (2001), Verhoef et al. (2004). Not many travel behaviour studies focus on heterogeneity of travelers, hence the aim of this thesis is to investigate different user-classes in road pricing problem (see e.g. Chen & Bernstein (2003)).

A more detailed graphical interpretation of the DTA framework is given in Figure 6.2. The proposed modeling framework contains two main components:(1) a travelers' behavior model (dynamic path and departure time choice model) and (2) a dynamic network loading (DNL) model. As can be seen from Figure 6.2, input to the DTA model are a set of available paths, the total travel demand, link travel time functions, and user-class specific parameters.

The first part of the DTA model is the simultaneous path and departure time model in which all travelers are distributed on available routes and departure times such that a dynamic stochastic user equilibrium (DSUE) will result. A stochastic instead of deterministic equilibrium is adopted because it is a more realistic approach and a more general statement of equilibrium than the deterministic equilibrium conditions (Sheffi, 1985). Namely, travelers choose their subjective optimal (often cheapest) route and departure time from a set of available routes and departure times depending on generalized travel costs. The choices are subjective because each traveler may perceive these costs differently. The generalized travel cost function and the DSUE will be described and discussed in more detail in Sections 5.4.1 and 5.4.2.

The second part of the general framework of the DTA model is the DNL model in which all flows are propagated along assigned routes for each departure time period. This part of the model simulates traffic through the network resulting in link travel times, which depend on the link densities. For more information about the DNL model used in this research see Section 6.5.

Note that the route and departure time choice model and the DNL model are interrelated. The link travel times depend on the route flows, while the route flows depend on the link travel times and tolls (included in the generalized travel costs).

The relation between the upper and lower level is as follows. Dynamic tolls from the pricing level affect the generalized travel costs (on which is the travel behavior based). These tolls will yield network conditions that have an impact on the policy objectives of the road authority. The upper and lower level components are interrelated to find the solution to the DOTD problem.

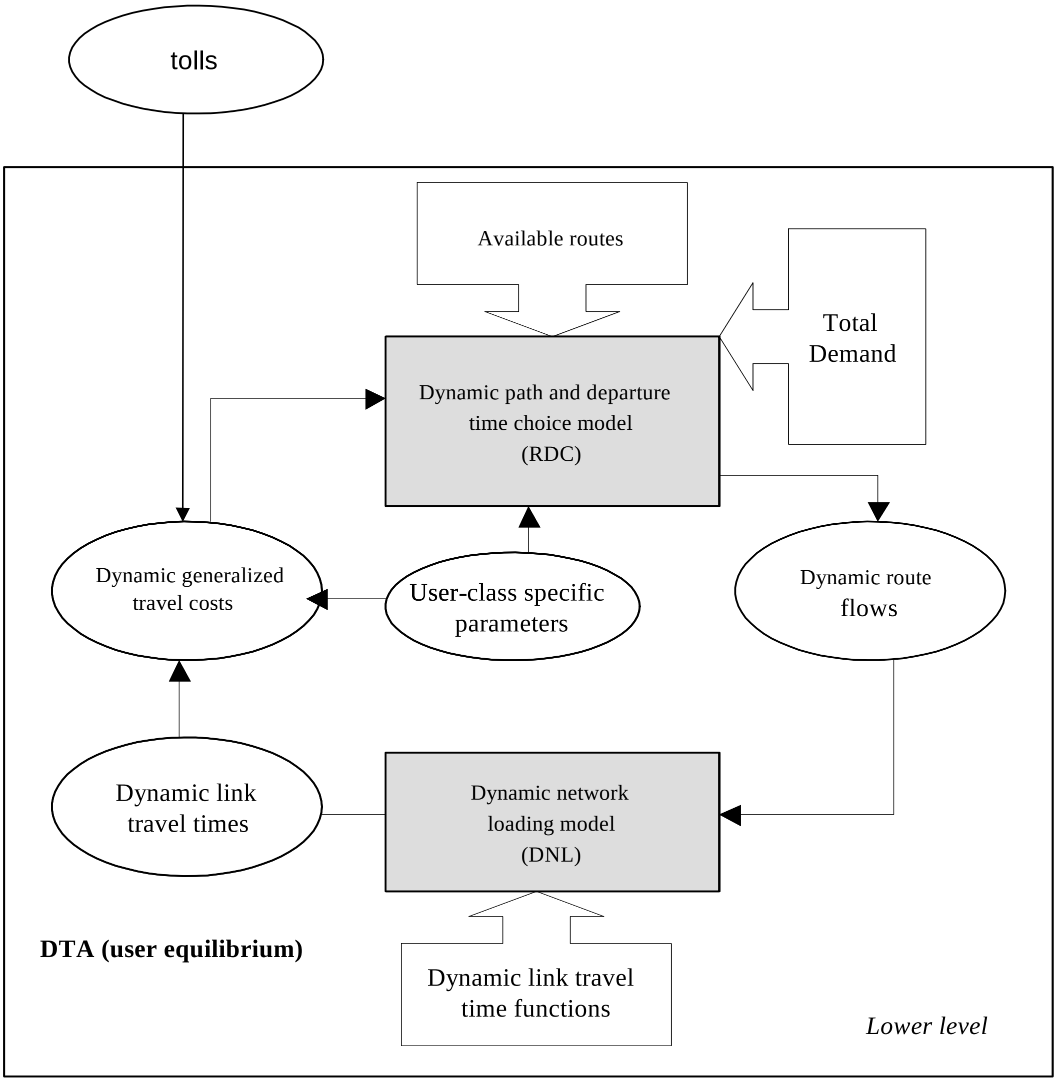

Figure 6.2: Framework of the proposed DTA modeling to handle the road pricing problem

6.4 Travel behavior model for road pricing

6.4.1 Specification of the generalized travel cost function to capture road pricing

An appropriate *random utility discrete choice model* will be used to describe route and departure time choice behavior of travelers. The dynamic generalized travel cost function of the existing models usually only consists of the dynamic travel time element. In general, different structures are possible for modeling the *joint* choice of route and departure time. One of the possibilities is given in Cascetta (2001) and we rewrite here the utility $U_{pm}^{rs}(k)$ of route p (from r to s) at departure time k for user-class m in the following expression:

$$U_{pm}^{rs}(k) = V_{pm}^{rs}(k) + \varepsilon_{pm}^{rs}(k) \tag{6.1}$$

where the total utility $U_{pm}^{rs}(k)$ is equal to the sum of observed utilities $V_{pm}^{rs}(k)$ of route p (from r to s) at departure time k for user-class m and unobserved utilities $\varepsilon_{pm}^{rs}(k)$ of route p (from r to s) at departure time k for user-class m.

The (observed) utility $V_{pm}^{rs}(k)$ of choosing route p and departure time k can be expressed in the following way:

$$V_{pm}^{rs}(k) = \overline{V}_{m}^{rs} - c_{pm}^{rs}(k) \tag{6.2}$$

where $\overline{V}_{m}^{rs}$ is the utility of spending time at destination s (work, leisure, etc.) departing from origin r for user-class m while $c_{pm}^{rs}(k)$ is the user-class m *generalized travel cost* for route p between origin r and destination s when departing at time interval k.

Since we also assume that this utility $\overline{V}_{m}^{rs}$ does not depend on the route or departure time, it is fixed for all routes and departure times, hence does not influence travel behavior. Therefore, in the remainder we will assume that $\overline{V}_{m}^{rs} = 0$, hence utility maximization implies cost minimization, such that the utility only consists of the disutility in terms of generalized travel costs. This implicitly means that trip choice is ignored, such that the travel demand is assumed to be fixed (which is different from the game theory analysis in Chapter 3).

The dynamic user-specific generalized travel cost $c_{pm}^{rs}(k)$ should include an element for the (dynamic) road-pricing toll. In addition, it is necessary to include penalties for deviating from the preferred arrival time (PAT) and preferred departure time (PDT) in the generalized travel cost function (Arnott et al. (1990)). In order to capture heterogeneity of users, different values for the value of time (VOT) as well as value of schedule delay (VOSD) are required. In other words, users with a low value of time and schedule delay are supposed to be more 'sensitive' to increases of generalized travel costs (where tolls are a part of perceived travel costs), such that they have a stronger response in travel behavior than users with a high value of time and schedule delay. Hence, users with a high value of time and schedule delay may afford to pay more for a less congested trip and a more preferred departure time. Here, we would like to refer to Van Amelsfort et al. (2005a) where behavioral responses to road pricing (based on stated preference data) are analyzed.

As a result, the dynamic user-specific generalized travel cost function $c_{pm}^{rs}(k)$ is expressed in the following way[1]:

$$c_{pm}^{rs}(k) = \alpha_m \tau_{pm}^{rs}(k) + \theta_{pm}^{rs}(k) + \beta_m \left|\left(k - \mathrm{PDT}_m^{rs}\right)\right| + \gamma_m \left|\left(\left(k + \tau_{pm}^{rs}(k)\right) - \mathrm{PAT}_m^{rs}\right)\right| \tag{6.3}$$

The dynamic generalized travel cost function (expression (6.3)) has the following components:

1. dynamic travel time $\tau_{pm}^{rs}(k)$ for path p between origin r and destination s for traveler class m departing at time k, together with user-specific value of time α_m;
2. dynamic tolls $\theta_{pm}^{rs}(k)$ for path p between origin r and destination s for traveler class m departing at time k;
3. penalty for deviating from PDT_m^{rs} together with the value of schedule delay β_m;
4. penalty for deviating from PAT_m^{rs} together with the value of schedule delay γ_m;

Parameters PDT_m^{rs} and PAT_m^{rs} are the origin-destination specific preferred departure and preferred arrival times for different users. The specification and calibration of these generalized travel cost functions is subject of research in a parallel study (see Van Amelsfort, 2005b). The departure time terms in the generalized travel cost function (6.3) are adopted from Vickrey (1969). It should be noted that, because of simplicity, we assume symmetric schedule delays costs with respect to the PDT and PAT, respectively.

The *dynamic route travel time* $\tau_p^{rs}(k)$ is additive by nature, i.e. it can be written as a sum of consecutive dynamic link travel times $\tau_{am}(t)$ for link a for traveler class m departing at time t:

$$\tau_{pm}^{rs}(k) = \sum_{a\in p}\sum_{t\in T}\delta_{pam}^{rs}(k,t)\cdot\tau_{am}(t) \tag{6.4}$$

where the dynamic route travel time $\tau_{pm}^{rs}(k)$ is computed using the dynamic link travel times $\tau_{am}(t)$ that belong to a specific route indicated by the *dynamic path-link incidence indicator* $\delta_{pam}^{rs}(k,t)$. This indicator is defined as follows:

$$\delta_{pam}^{rs}(k,t) = \left\{\begin{array}{ll} 1, & \text{if travelers departing at time } k \text{ along route } p \text{ reach link } a \text{ at time } t, \\ 0, & \text{otherwise} \end{array}\right\} \tag{6.5}$$

In general, the dynamic path tolls $\theta_{pm}^{rs}(k)$ need not have this additive cost structure. Therefore, the path choice model in the DTA model (Figure 6.2) should be able to handle

[1]The use of β and γ deviates from the conventional notation where β is shadow cost of early arrivals, and γ of late arrivals. Also, in our modeling appraoch, we use symmetric shadow costs.

this. In case the dynamic route tolls are additive (e.g. a road pricing strategy where travelers are independently tolled each time they enter a new road segment), then the dynamic route tolls $\theta_{pm}^{rs}(k)$ can be computed by adding the appropriate dynamic link tolls $\theta_{am}(t)$ for link a and traveler class m at time t, i.e.:

$$\theta_{pm}^{rs}(k) = \sum_{a \in p} \sum_{t \in T} \delta_{pam}^{rs}(k,t) \cdot \theta_{am}(t) \tag{6.6}$$

With $\theta_{am}(t)$ we denote tolls that traveler class m pays upon entering of tolled link a at time t .

It should be noted that the tolls in this modeling framework have to be paid at link entrance (hence each traveler will pay the toll value once upon entering the tolled link, see Chapter 2). If travelers have to pay when they exit the link, the tolls are still additive but Equation (6.6) has to be adapted.

6.4.2 Dynamic stochastic user equilibrium conditions

We will present the dynamic stochastic and multi-user class extension of **Wardrop's first principle** to include departure time choice of the travelers:

Definition 10 *For each user-class and for each origin-destination (OD) pair, the perceived general travel costs for traveling between a specific OD pair are equal for all used routes and departure times, and less than (or equal to) the generalized travel costs which would be perceived by that user-class for any unused departure time and unused feasible route.*

In this research a *stochastic* user equilibrium is used meaning that the travel costs are perceived costs. The perceived travel costs are random variables with certain distributions. In other words, the probability that a route and departure time is chosen among the set of available routes and departure times is equal to the probability this route and departure time is perceived to have minimum generalized travel costs. It should be noted that Wardrop's condition holds separately for each user-class. More information about the stochastic equilibrium conditions can be found in He (1997).

6.4.3 Route and departure time choice models

Here we concentrate on the travelers' behavior model, which is jointly modeling route and departure time choice. Various specifications of a random utility model can be derived by assuming different joint probability distribution functions for the random residuals $\varepsilon_{pm}^{rs}(k)$ in expression (6.1). For more details about choice models we refer to Cascetta (2001). If the random residuals $\varepsilon_{pm}^{rs}(k)$ are independently and identically *Gumbel distributed* then the choice probability of any alternative depends on the differences between the systematic utilities of all other alternatives and leads to the well-known multinomial logit (MNL) model. For more information see e.g. McFadden (1974), Mahmassani & Herman (1984).

In this work, a path-size (PS) logit model is used (see more in e.g. Hoogendoorn-Lanser (2005), Ramming (2001)). A path-size logit model is a special case of the multinomial logit (MNL) model. A PS model approximates the amount of overlap of path p with all other alternatives in the set of available paths. The joint probability $\zeta_m^{rs}(p,k)$ that path p is chosen from the set of available paths as well as departure times k from the set of available departure times between origin r and destination s for travelers class m is given by:

$$\zeta_m^{rs}(p,k) = \frac{\xi_p^{rs} \exp\left(-\mu c_{pm}^{rs}(k)\right)}{\sum\limits_{\overline{p}\in P^{rs}} \sum\limits_{\overline{k}\in K} \xi_{\overline{p}}^{rs} \exp\left(-\mu c_{\overline{p}m}^{rs}(\overline{k})\right)} \tag{6.7}$$

where μ is a *scale parameter* of the utilities in the joint path-size model at the route and departure time choice level, while ξ_p^{rs} is the path-size of path p ($0 \leq \xi_p^{rs} \leq 1$). If path p is unique, its path-size ξ_p^{rs} is equal to 1, and thus the disutility of this path is unchanged, simplifying into the MNL model.

The extent to which a link a contributes to the PS-factor ξ_p^{rs} depends on the number of paths using this link. The simplest form of ξ_p^{rs} is given by:

$$\xi_p^{rs} = \sum_{a\in p} \left(\frac{l_a}{L_p}\right) \frac{1}{N_a} \tag{6.8}$$

where l_a is the length of link a, L_p is the length of path p, while N_a is the number of alternative routes from r to s using link a. For more information about path-size formulations, see e.g. Hoogendoorn-Lanser (2005).

Given the class-specific travel demand rates D_m^{rs} between origin r and destination s for user-class m, then the dynamic class-specific path flow rates $q_{mp}^{rs}(k)$ of path p between origin r and destination s at time k for traveler class m can be determined by the following expression:

$$q_{pm}^{rs}(k) = \zeta_m^{rs}(p,k) \cdot D_m^{rs}, \qquad \forall (r,s) \in RS,\ \forall p \in P^{rs}, \forall k \in K,\ \forall m \in M \tag{6.9}$$

6.4.4 VI problem formulation of the DTA for road pricing

As stated before, the lower level of the DOTD problem can be converted into an equivalent variational inequality (VI) problem (see e.g. Bliemer & Bovy (2003), Chabini (2001)).

Therefore, for the DSUE principle defined in Section 6.4.2, an appropriate VI problem formulation is used.

We aim to find a *dynamic path flow pattern* $\overline{q}_{pm}^{rs}(k,\theta) \in \Psi$ such that

$$\sum_{rs\in(RS)}\sum_{p\in P^{rs}}\sum_{k\in K}\sum_{m\in M}\overline{Q}_{pm}^{rs}(k,\theta)\cdot\left[q_{pm}^{rs}(k)-\overline{q}_{pm}^{rs}(k,\theta)\right]\geq 0, \qquad \forall q_{pm}^{rs}(k)\in\Psi \quad (6.10)$$

where Ψ is defined as the set of all feasible path flow rates $q_{pm}^{rs}(k)$ satisfying the following constraints (6.11) and (6.12):

(flow conservation constraints)

$$\sum_{k\in K}\sum_{p\in P^{rs}} q_{pm}^{rs}(k) = D_m^{rs} \qquad \forall (r,s)\in RS,\ \forall m\in M \quad (6.11)$$

(nonnegativity constraints)

$$q_{pm}^{rs}(k)\geq 0, \qquad \forall (r,s)\in RS, \forall p\in P^{rs}, \forall k\in K,\ \forall m\in M \quad (6.12)$$

In Equation (6.10), the term $\overline{Q}_{pm}^{rs}(k,\theta)$ is some 'perceived equilibrium cost' (multiplication of path flow rates and a cost derivative) on path p for user class m between an origin r and destination s, and is given by:

$$\overline{Q}_{pm}^{rs}(k,\theta) = \left[\overline{q}_{pm}^{rs}(k) - D_m^{rs}\zeta_m^{rs}(p,k)\right]\cdot\frac{\partial\overline{c}_{pm}^{rs}(k,\theta)}{\partial q_{pm}^{rs}(k)} \quad (6.13)$$

In Equation (6.13), the term $\dfrac{\partial\overline{c}_{pm}^{rs}(k,\theta)}{\partial q_{pm}^{rs}(k)}$ is evaluated in $\overline{q}_{pm}^{rs}(k,\theta)$. More about this perceived equilibrium travel cost can be found in He (1997).

6.5 DNL component of the proposed DTA model

The dynamic network loading model (DNL) component is formulated as a system of equations expressing link dynamics, flow conservation, flow propagation and boundary constraints.

The dynamic network loading (DNL) component 'simulates' the route flows on the network, yielding link flows, link volumes, and link travel times. The DNL model used in this research is a very simple system of equations adapted from Chabini (2001) and Bliemer et al. (2004), in which the flow propagation equation is simplified by assuming that there are no subintervals within one time interval and that the link travel time is stationary. In this case, the equations are similar to the ones proposed by Ran & Boyce (1996).

Because of simplicity we assume the same travel time for all user-classes m in the proposed DNL model, hence $\tau_{am}(t) = \tau_a(t), \ \forall m \in M$. It should be noted that a class-specific DNL formulation can be easily derived (see Bliemer et al. (2004)).

The following set of equations describes the DNL model:

$$v_{ap}^{rs}(t + \tilde{\tau}_a(t)) = u_{ap}^{rs}(t) \tag{6.14}$$

$$u_{ap}^{rs}(t) = \left\{ \begin{array}{ll} \sum\limits_m q_{pm}^{rs}(t) & \text{if } a \text{ is the first link on path } p \\ v_{a'p}^{rs}(t) & \text{if } a' \text{ is the previous link on path } p \end{array} \right\} \tag{6.15}$$

$$u_a(t) = \sum_{(r,s)} \sum_{p \in P^{rs}} u_{ap}^{rs}(t) \tag{6.16}$$

$$v_a(t) = \sum_{(r,s)} \sum_{p \in P^{rs}} v_{ap}^{rs}(t) \tag{6.17}$$

$$x_a(t) = \sum_{w \le t} (u_a(w) - v_a(w)) \cdot \Delta t \tag{6.18}$$

$$\tau_a(t) = g_a(x_a(t)) \tag{6.19}$$

The **flow propagation** equations in Equation (6.14), which describe the propagation of the inflows $u_{ap}^{rs}(t)$ through the link and therefore determine the outflows v_{ap}^{rs}, relate the inflows and outflows of link a at time interval t of vehicles traveling on route p from r to s, respectively. This equation simply states that traffic that enters link a at time t will exit the link when the link travel time elapses. Note that since we are dealing with a discrete-time problem, the link exit time needs to be an integer value. Therefore, $\tilde{\tau}_a(t)$ is used, which simply rounds off the travel time (expressed in time intervals) to the nearest integer. This flow propagation equation (Equation (6.14)) is valid only if stationary link travel times are assumed. More appropriate models can be used (Chabini (2001); Bliemer & Bovy (2003); Astarita (1996)) but for our purpose the proposed flow propagation equation is suitable. It should be noted that the First-In-First-Out (FIFO) condition is not explicitly assumed in our model, however FIFO violations could be detected afterwards. The FIFO condition states that if a vehicle enters a link a at time t, the other vehicles which enter the link later than that vehicle will also leave the link later than that vehicle.

Equation (6.15) describe the **flow conservation** equations. If link a is the first link on a route, the inflow rate is equal to the corresponding route flows determined by the simultaneous route and departure time choice model. Since we have assumed that all vehicles travel at the same speed, the low and high VOT users can be combined in the DNL model by summing them up. If link a is not the first link on a route, then the link inflow rate $u_{ap}^{rs}(t)$ is equal to the link outflow rate $v_{a'p}^{rs}(t)$ of the previous link.

Equations (6.16)–(6.18) are **definitions.** The first two simply stating that the total link inflows (or outflows) are determined by adding all link inflows (or outflows) for all routes that flow into (out of) link a at that time interval. Equation (6.18) defines the number of vehicles $x_a(t)$ on link a at the beginning of time interval t, which is by definition equal to the total number of vehicles that have entered the link until time interval t, minus the total number of vehicles that have exited the link. The cummulative inflows and outflows are computed by multiplying the appropriate flow rates by the time inteval size, Δt. Finally, Equation (6.19) relates to the travel time on that link as a non-decreasing, link-specific function $g_a(\cdot)$ (where the travel time depend on the number of vehicles, $x_a(t)$).

6.6 Summary and Conclusions

In this chapter we have analyzed how the behavioral part of the DOTD problem might be extended to road pricing. With behavioral part of the DOTD we assume a typical equilibrium problem where different travelers' choices can be considered. For that purpose a DTA model consisting of a travelers' choices model and a dynamic network loading can be used. In this chapter DTA problem formulations are stated where the VI problem formulation is chosen as the most suitable for modeling of the behavioral part of the DOTD problem. The dynamic stochastic user equilibrium (DSUE) principle is used in this work. In addition, travel choices are modeled using the path-size model. A DNL model component of the proposed DTA model formulated as a system of equations expressing link dynamics, flow conservation, flow propagation and boundary constraints is outlined in this chapter.

In the course of this analysis, we have demonstrated how the dynamic modeling framework of the multi-class DTA problem with road pricing is formulated. The main contribution is given to the derivation of the dynamic generalized travel cost function to capture road pricing. The generalized travel cost function is modified to capture dynamic tolls and heterogeneity of travelers.

The work done here can be extended in various ways. For example, the dynamic generalized travel cost function can be further developed (e.g. to include the risk or reliability aspect) which will be the subject of the future research. Also, more sophisticated DNL models can be used.

Part IV

Computational experiments

Chapter 7

Computational experiments on ‘small-networks’

7.1 Introduction

The aim of this chapter is to show the adequacy of the proposed DOTD model as well as to prove the correctness and feasibility of the procedure to solve the DOTD problem proposed in this thesis. In Chapter 2, the DOTD model framework has been presented. Furthermore, the discrete-time multi-class optimal toll design problem as well as extensions of existing dynamic traffic assignment (DTA) model to road pricing have been formulated in Chapters 5 and 6, respectively.

In this chapter, a few case studies for the proposed DOTD problem are presented. In these case studies, the DOTD problem is considered from different modeling perspectives with regard to network specifications, travel choice of travelers, policy objectives, user-class specifications and toll schemes. Because of increasing modeling and computational complexity, all experiments are performed on a few simple hypothetical transportation networks.

The following questions will be answered in this chapter. What is our motivation to apply different case studies on the DOTD problem? How do the objective functions for the DOTD problem look like? How and in which ways do the imposed tolls impact the travel patterns of different user classes in the proposed DOTD problem?

The expected results of this chapter are to give more insights into the *model’s properties* of the DOTD problem. Moreover, characteristics of the objective function(s) of the road authority are shown. Furthermore, we examine the impact of tolls on travel patterns of different classes of travelers due to changes in route and departure time choice (in hypothetical networks).

The outline of this chapter is as follows. First, different toll patterns adopted in the experiments are described in Section 7.2. An overview of the conducted case studies with a motivation for choices is given in Section 7.3. After that, we start with the most simple case, by examining a simple corridor network in Section 7.4. In this section, the travel demand and supply side for the corridor network are shown as well as results of the DOTD problem with departure time choice. Several case studies including classes of travelers with different values of time and schedule delays are presented. The second case study considers a single-OD, two-route network (analyzed in Section 7.5), where also route choice of travelers is analyzed. Different tolling schemes (uniform, (quasi)uniform and variable) are examined in this case study. The impacts of tolls on different user groups are analyzed. Finally, we present a multi-OD, multiple route network case in Section 7.6. In this case study, the adequacy of proposed DOTD model is shown on a multi-OD network, where two links are simultaneously tolled. In all cases policy objectives of maximizing revenues and minimizing travel time are considered. This chapter finishes with a conclusion part and further recommendations (Section 7.7).

7.2 Toll patterns adopted in the experiments

Different toll patterns can be formulated in the upper level of the DOTD problem. More about different toll patterns can be found in Chapter 2. On the one hand, tolls can be introduced as uniform tolls, constant over all time periods for a specific link a. On the other hand, tolls can be variable over time periods on a specific link a.

For each tolled link a, $\forall a \in Y$ (with Y we denoted a set of all links on a transportation network which can be tolled) there are at least three options:

- uniform toll fare

$$\theta_a(t) = \overline{\theta}_a, \quad \forall t \in T, \quad \forall a \in Y \tag{7.1}$$

  Toll values are constant over all time periods t but possibly different over paths p. With this toll pattern, the travelers have a route choice possibility (if alternative routes exist). However, because all time periods are tolled equally, departure time choice is absent.

- (quasi)uniform toll fare

$$\theta_a(t) = \left\{ \begin{array}{ll} \overline{\theta}_a & \forall t \in T^{'}, \ \forall a \in Y \\ 0, & \text{otherwise} \end{array} \right\} \tag{7.2}$$

  In this tolling pattern, constant tolls $\overline{\theta}_a$ are charged only in a subset of time periods $T'(T^{'} \subset T)$, while during other time periods tolls are set to zero. In this way, the travelers have an opportunity to choose non-tolled time periods (departure time choice).

- variable toll fare

$$\theta_a(t) = \left\{ \begin{array}{ll} \phi_a(t) \cdot \overline{\theta}_a, & \forall t \in T^{'}, \ \forall a \in Y \\ 0, & \text{otherwise} \end{array} \right\} \tag{7.3}$$

  where $\phi_a(t)$ is a given predefined function for link a over time t and $\overline{\theta}_a$ is the maximal toll value resulting from the calculation. For the predefined function $\phi_a(t)$ the following holds:

$$0 \leq \phi_a(t) \leq 1 \tag{7.4}$$

  Clearly, in this tolling pattern, variable tolls $\theta_a(t)$ can have different fare values in different time periods. With this toll pattern, the travelers have a chance to choose the route (if alternative routes exist) and time period in which they will depart, depending on their personal travel preferences.

Expression (7.1) is a special case of (7.3) if $\phi_a(t) = 1$ and all time periods are tolled (thus $T = T'$). Expression (7.2) is also a special case of Expression (7.3), where $\phi_a(t) = 1$ on a subset of tolled periods T'. In general, all three tolling options can be expressed by (7.3).

For more elaborate pricing measures see Chapter 2.

7.3 Experimental set-up of the DOTD problem

In this section the experimental set-up of our test of the DOTD model is established as well as assumptions and specifications. The aim of the experimental set-up of our test is to give more insight into the DOTD problem from different modeling perspectives. Different case studies are performed with different criteria aiming to show the adequacy of the proposed DOTD model as well as the correctness and feasibility of the proposed mathematical model. On the one hand, with the adequacy of the model we assume that the model describes the real problem in an appropriate way (including below mentioned assumptions). On the other hand, with the correctness and feasibility we assume that the model is able to solve the real problem within reasonable boundaries (e.g. time and memory consumption) and produces plausible results.

In the experiments, the following assumptions are made:

- In all experiments *heterogeneous users* are taken into account with different parameters for time and cost attributes;
- A given and fixed *total travel demand* is assumed, meaning that the total number of travelers on the network is fixed (but variable per time period and over space); the motivation to consider fixed total demand is because this research considers

mainly mandatory trips (and not optional trips); according to Ubbels (2006) and MuConsult (2002) commuting trips are hard to replace; more explanation can be found in Chapter 6);

- Only *route and departure time choice* of travelers are taken into account, the other travel choices are neglected in experiments; nevertheless, the no-trip or no-car trip choice can be easily included in this modeling framework (see Chapter 8);
- *Discrete time* formulation is used in experiments;
- The toll locations and tolled time periods are input into the model, only the *toll levels* are to be determined.

Specifications of the experimental setup are made with regard to different criteria:

1. Network specification (single-OD network with only one route, single OD network with dual routes, or multi-OD network with multiple routes);
2. Travel choice of travelers (route and/or departure time choice);
3. Policy objectives of the road authority (e.g. optimization of total travel time or revenue generation);
4. Toll schemes (uniform, (quasi) uniform and variable) fare based on passage of travelers when entering the tolled link (see more about different tolling schemes in Chapter 2);
5. User class specifications with respect to value of time (VOT) and value of schedule delay (VOSD).

The experimental set-up of our test of the DOTD problem is outlined in Table 7.1, where a range of experiments are classified regarding test network, travel choices, policy objectives, number of links tolled, and user-class specifications. For all experiments, a zero-toll reference case is calculated as well.

The motivation to select the set of experiments given in Table 7.1 among all possible experiments is as follows. We start by exploring the simplest case, that is a so-called 'corridor' network. A corridor network is a network with one (or several) links constituting a single route. Clearly, only departure time choice can be analyzed in such a network (route choice is absent). Hence, in these case studies ($E1 - E4$ in Table 7.1) we analyze departure time choice only. The aim of case studies on a corridor network is to prove the correctness of the road pricing model (proposed in Chapter 5) and departure time choice model (proposed in Chapter 6). With the correctness of the model we consider that the model is able to solve the problem within reasonable boundaries and produces plausible results.

Table 7.1: Experimental set-up of all tolling case studies given in this thesis

	test network	travel choices	policy objective	tolled link X tolling regime	user-class spec.
$E1$			REV		
$E2$			TTT		VOT
$E3$	corridor	DTC	REV	1 x var	VOT +
$E4$			TTT		VOSD
$E5$				1 x uni	
$E6$	dual		REV	1 x (q)uni	
$E7$	route	DTC+RC		1 x var	VOT
$E8$	network			1 x uni	
$E9$			TTT	1 x (q)uni	
$E10$				1 x var	
$E11$	'Chen'		REV	2 x var	
$E12$	network	DTC+RC	TTT		VOT

Legend:

corridor	single-OD, single route network
dual network	single-OD, dual route network
'Chen' net	multiple-OD, multiple route network
DTC	departure time choice
RC	route choice
TTT	total travel time minimization
REV	revenue maximization
uni	uniform tolls
(q)uni	quasi uniform tolls
var	variable tolls
VOT	value of time
VOSD	value of schedule delay

In those case studies ($E1-E4$), our aim is to specially outline departure time choice of travelers. Four different case studies are presented with regard to different policy objectives to be reached (revenues maximization and total travel time minimization) and with regard to the heterogeneity of travelers (with value of time and schedule delay differences). Different groups of travelers, namely with low value of time and schedule delay and with high value of time and schedule delay are considered and analyzed in these experiments. Because of the general aim to consider variable tolls in this thesis, as well as corridor network's characteristics (namely, travelers have no route choice at all) we show only *variable* tolls in this case study. In other words, with variable tolls the travelers have the opportunity to change their departure times.

A more realistic case to consider is a dual-route network with one origin destination pair where only one route is tolled (experiments $E5-E10$). Other than in experiments $E1-E4$, departure time choice as well as route choice of travelers can be analyzed in this type of network. In these case studies we focus more on effects of different tolling schemes (uniform, quasi(uniform) and variable) applied to the DOTD problem. Experimental setup for the single OD, dual-route network is presented in Table 7.1.

In case studies $E11$ and $E12$ (Table 7.1), a multiple-OD network with multiple routes is considered. Case studies where multiple links may be tolled are especially interesting from a policy objective point of view. The focus in this case study is to investigate different possibilities of tolling different links in order to optimize network performance according to a given policy objective.

In all experiments, a dynamic traffic assignment (DTA) model is adapted for pricing design (explained in Chapter 5). A dynamic stochastic multi-user equilibrium (DSUE) demand model is used to capture the behavioral part of the DOTD problem. Generalized travel cost functions as well as logit and path-size models are used to describe the route and departure time choice of travelers. More details about the DTA model can be found in Chapter 6 of this thesis. The solution to the DOTD problem in all case studies is obtained using a simple grid search method. The *dynamic duality gap function* to obtain the convergence of the proposed models is used.

In the following sections, the proposed case studies (Table 7.1) will be presented and explained in more detail.

7.4 Case studies on a corridor network ($E1-E4$)

In this section, the case studies $E1-E4$ (outlined in Table 7.1) on a corridor network are presented. The policy objectives of the road authority to be considered in these case studies are twofold: revenue maximization and total travel time minimization, respectively. First, a description of a corridor network including link travel time functions is given. After that, the parameters of the corridor network (demand input) are specified. The zero-toll reference situation is given showing the results of the DTA with respect to

Figure 7.1: The route between Schiedam and Hoogvliet

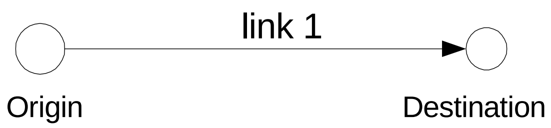

Figure 7.2: Network description of a corridor network

path flow rates and costs. Then, the toll pattern is introduced. Results show values of both policy objectives as well as corresponding optimal toll patterns, path flows and path costs. Case studies $E1$ and $E2$, with different groups of travelers regarding VOT as well as case studies $E3$ and $E4$ including groups with different VOSD sensitivities are described in this section. It should be noted that the zero-toll reference case for cases $E1$and $E2$ (only VOT) are different from one with VOT and VOSD ($E3$ and $E4$). A discussion of the presented results is given and conclusions are derived.

7.4.1 Description of a corridor network (the supply part of the DOTD problem)

A most simple network to consider in the road pricing problem is the corridor network. However, from these most simple cases one can learn a lot about DOTD model's properties. In practice, this network can represent, e.g. a (isolated) freeway route ($A4$) between two small Dutch cities: Schiedam and Hoogvliet in a densely populated Randstad area in the Netherlands (Figure 7.1).

As can be seen from Figure 7.2, this network comprises of only one link connecting a single origin-destination pair. The mathematical notation used to describe this network is already introduced in Chapter 5 of this thesis. Experiments $E1 - E4$ in Table 7.1 have been applied to this corridor network (Figure 7.2).

Because we use a discrete time formulation, we need to divide the whole studied time

Table 7.2: Link travel time function parameters for the corridor network

	parameter	notation	value
1.	free flow time	τ^0	10.5 [min]
2.	impact of $x(t)$	b	0.00005 [min/veh^2]
3.	power of $x(t)$	c	2

period T into a number of smaller time intervals with length Δ, indexed by t. The length of an interval must be significally smaller than the free flow travel time on the network, while time period T is assumed to be long enough such that all travel demand enters and exits the network. In that way, flow dynamics of each traveler can be captured and properly analyzed within given time scope. In these experiments we use an interval Δ of 3 minutes.

Link travel time functions

A dynamic link travel time functions (explained in Chapter 6, Equation (6.19)) denoted by $\tau(t)$, is used in this corridor network.

$$\tau_a(t) = \tau_a^0 + b_a x_a^{c_a}(t) \tag{7.5}$$

where $x_a(t)$ the number of vehicles on link at the beginning of time interval t (which is by definition equal to the total number of vehicles that have entered the link until time interval t, minus the total number of vehicles that have exited the link). The parameters τ_a^0, b_a and c_a in Equation (7.5) describe the free flow travel time on the link, and impact of the number of vehicles on the link $x(t)$, respectively. The values of parameters of the dynamic link travel time function of the corridor network are given in Table 7.2.

Our aim is to analyze the corridor network conditions (travel time rates, flow rates, route costs) as well as impact of tolls on travel choices of different groups of travelers with regard to specifications for experiments $E1-E4$ given in Table 7.1.

7.4.2 Travel demand input

In this experiment, the total travel demand over the whole studied period T is $D^{rs}_{total} = 6600$, equally divided between two groups of travelers, $D^{rs}_1 = 3300$ and $D^{rs}_2 = 3300$. In the first set of case studies $E1$ and $E2$, the travelers are divided into two different groups regarding their differences in VOT only (with low and high VOT): $\alpha_1 = 0.08$ [eur/min] for the group with low VOT and $\alpha_2 = 0.25$ [eur/min] for the group with high VOT. (Expressed in hours, $\alpha_1 = 5$ [eur/h] for travelers with low VOT while $\alpha_2 = 15$ [eur/h] for travelers with high VOT).

Table 7.3: Parameters for the corridor network: demand side

	parameter	*notation*	*value*	*unit*
1.	preferred departure time	PDT	10	Δt
2.	preferred arrival time	PAT	20	Δt
3.	number of departure time intervals	K	40	-
4.	penalty for deviation from PDT (for both groups)	β	0.08	eur/min
5.	penalty for deviation from PAT (for both groups)	γ	0.33	eur/min
6.	VOT group 1	α_1	0.08	eur/min
7.	VOT group 2	α_2	0.25	eur/min
8.	scale parameter (logit model)	μ	0.35	-

The general dynamic route cost function, introduced in Chapter 6, Equation (6.3), and stated here again, is used in these experiments:

$$c_{pm}^{rs}(k) = \alpha_m \tau_{pm}^{rs}(k) + \theta_{pm}^{rs}(k) + \beta_m \left|\left(k - \mathrm{PDT}_m^{rs}\right)\right| + \gamma_m \left|\left(\left(k + \tau_{pm}^{rs}(k)\right) - \mathrm{PAT}_m^{rs}\right)\right| \tag{7.6}$$

Input parameters for the dynamic route cost function in the corridor network are given in Table 7.3. A departure time logit choice model with scale parameter $\mu = 0.35$ is applied for proportions of travelers along 40 time periods Δt, where $\Delta t = 3$ [min] (Table 7.3). The logit choice model is introduced in Chapter 6, Equation (6.7).

In these experiments ($E1$ and $E2$) we suppose that the preferred departure time (PDT) and preferred arrival time (PAT) are the same for all travelers. In general, PDT and PAT are origin-destination specific (as can be seen in the last case studies $E11$ and $E12$). For more information about the generalized travel cost function with all these parameters, see Chapter 6, Equation (6.3). Using these travel demand parameters (Table 7.3), a stochastic multi-user equilibrium demand model (explained in Chapter 6) is used in these experiments.

7.4.3 Experiments on corridor network with groups of travelers with different VOT only ($E1$, $E2$)

Zero-toll situation

In order to decide which time periods to toll, and as a reference case an analysis of zero-toll situation is needed. The travel costs and corresponding flow pattern in different time periods are given in Figure 7.3.

From the non-tolled case (see Figure 7.3b), it can be seen that most travelers occur in time periods from 8 to 16, suggesting that these time periods should be tolled in order to spread travel demand over less congested time periods. According to the zero-toll analysis, it can

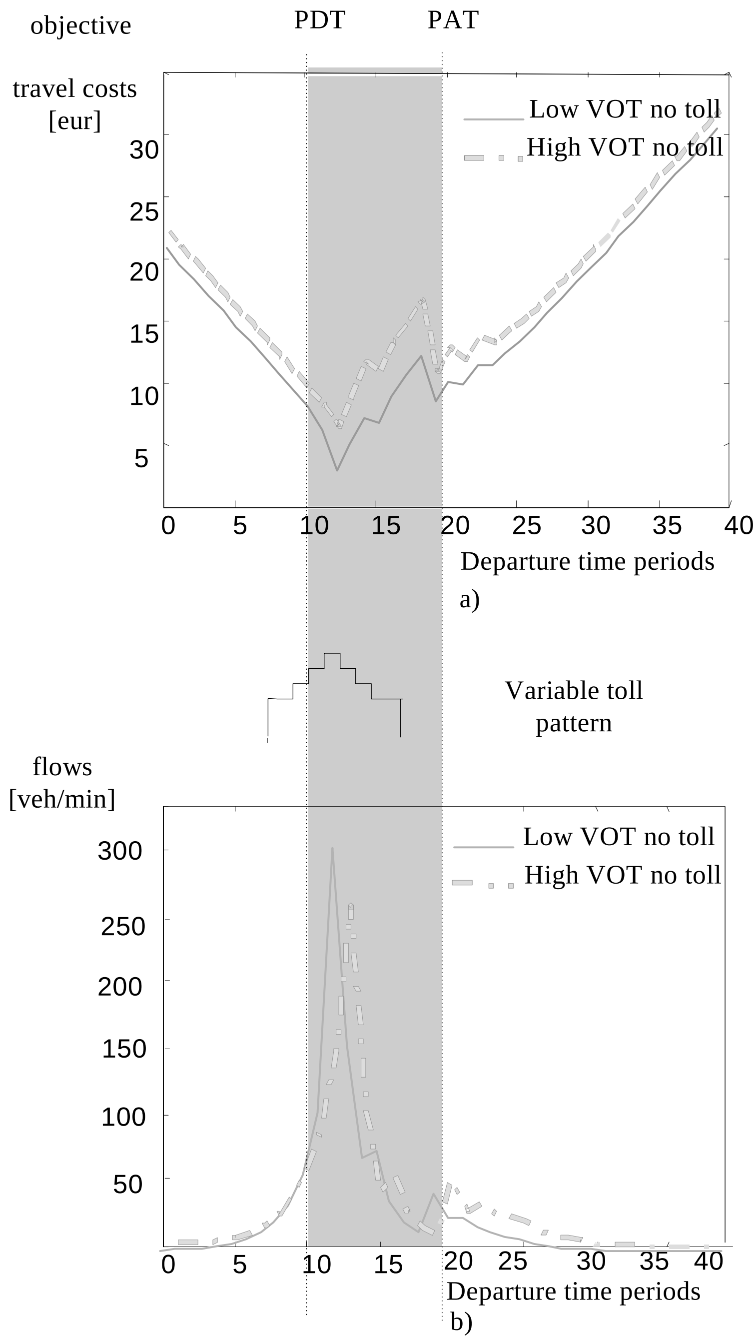

Figure 7.3: Temporal demand pattern and objective travel costs by user-class, no toll case

be seen that the travel demand in time periods 11, 12 and 13 is highest (peak period). It seems plausible therefore, to apply the highest tolls in these time periods (11, 12 and 13) while lower tolls might be imposed on the shoulders of the peak-time period (time periods 8, 9, 10, 14, 15 and 16). In time periods lower than 8 and higher than 16 no tolls are assumed to be needed.

From Figure 7.3a it can be noticed that the equilibrium route costs for both classes of travelers are not equal but significantly higher at the beginning and at the end of considered time period. The reason is that these time periods are far away from preferred departures and arrival times hence the value of schedule delay components of the route cost functions in these time periods become higher. Some travelers cannot deviate so much from their preferred departure or arrival times (especially in so-called mandatory trips). Fully equalized equilibrium costs will not result because of the stochastic demand predictions.

There exist also jumps in the route cost functions (see Figure 7.3a, departure time periods between 15 and 20). Actually, the route cost function is comprised of three cost components, travel cost, the curve representing arrival schedule delay and the curve for departure schedule delay. In Figure 7.3a these three curves are combined. The shapes of these curves follow the jumps in the cumulative route cost functions represented in Figure 7.3a.

High VOT people have always higher travel costs caused by their higher value of time component. Because this component is important (besides travel time) part of the dynamic route cost function, the higher value of time plays significant role and hence has impact on resulting dynamic route cost function. Clearly, this happens in this case without tolls. It will be interesting to analyze this phenomenon in the case where tolls are imposed (because tolls are also a significant part of the generalized dynamic route cost function).

It should be noted that if different values for PAT and PDT for travelers will be chosen, a (completely) different demand pattern will be obtained, because PAT and PDT are parts of the generalized travel cost function (see Chapter 6, Equation (6.3)) and have an important influence on travel costs of travelers.

The non-smooth cost pattern might also be caused by computational factors such as the time period discretization and insufficient convergence of the DTA.

Toll pattern

The no-toll temporal flow pattern given in Figure 7.3b will be the basis to determine the variable tolls corresponding to traffic flows (usually higher proportions of tolls for more congested time periods and vise versa). Furthermore, for variable tolls we assume there is a given predefined function $\phi(t)$ over time for the link temporal toll pattern. In these experiments, the proportions of this function $\phi(t)$ (see Section 7.2) for $E1$ and $E2$ are given in Table 7.4. In other words, the proportions of the time-varying tolls are fixed, hence the shape and the time distribution of the toll levels over time is given. A graphical

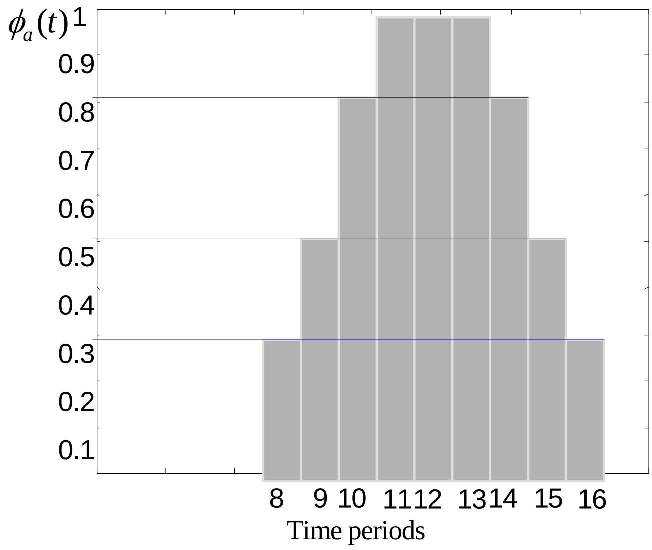

Figure 7.4: Assumed temporal fare pattern for the corridor network experiment

Table 7.4: Parameters for the corridor network: supply part

	parameters	*notation*	*value*	*unit*
1	minimal toll	$\theta_a^{\min}$	0	eur/passage
2	maximal toll level	$\theta_a^{\max}$	50	eur/passage
3	predefined function	$\phi_a(t)$	1, periods $11-13$ 0.8, periods 10 and 14 0.5, periods 9 and 15 0.3, periods 8 and 16 0, other time periods	-

description of the assumed variable tolling pattern over time to be applied to this corridor network is given in Figure 7.4.

As a result, there is only a single toll value $\overline{\theta}$ to be determined. Hence, $\overline{\theta}$ is a single (maximum) toll value to be derived on which (using predefined function $\phi(t)$, see Equation (7.3)) the temporal toll pattern $\theta(t)$ is determined as follows:

$$\theta(t) = \phi(t) \cdot \overline{\theta} \tag{7.7}$$

where

$$\theta^{\min} \leq \overline{\theta} \leq \theta^{\max} \ \text{ and } \quad 0 \leq \phi(t) \leq 1 \tag{7.8}$$

Toll parameters used in these experiments are given in Table 7.4.

Using the given input parameters (the minimal and maximal toll values ($\theta^{\min}$, $\theta^{\max}$), and given proportions ($\phi(t)$, Equation (7.3)) the following parameters should be determined: a single optimal toll value $\overline{\theta}$, and the resulting variable toll pattern $\theta(t)$.

It should be noted that this variable toll pattern approach with fixed proportions is only an approximation. The motivation was to spread toll values among time periods in a way that they can increase or decrease for a predefined value (see Equation 5.10). However, an ideal DOTD should produce optimal levels for all variable tolls at all available time periods. This will be the subject of the further study. A more realistic approach, of course, should be to apply dynamic tolls where the toll value in each time period directly depends on the flow pattern (see Chapter 8).

Case-studies ($E1$ and $E2$) with tolls

Maximizing total toll revenues- Experiment $E1$ In this experiment, tolls are imposed according to the selected time periods (see zero-toll situation) given proportions of the function $\phi(t)$ in Table 7.4.

Grid-search is used to determine the highest toll value, resulting in $\overline{\theta} = 9$ [eur]. Based on this result, the temporal toll pattern is determined (see Figure 7.5c). Thus, in time periods 11, 12, and 13 the highest fare value should be charged (9 eur/passage). In time periods 10 and 14 the fare is 7.2 [eur/passage] while in time periods 9 and 15 the fare is 4.5 [eur/passage]. The lowest tolls should be imposed in time periods 8 and 16, namely 2.7 [eur/passage].

The impacts of tolls are a strong peak reduction and a demand redirection towards less tolled time periods.

Trip costs including toll for both groups of travelers as well as for the tolled and non-tolled case are illustrated in Figure 7.5a. Equilibrium trip costs appear higher in the beginning and at the end of the time period. Again, that is because of effects of VOSD of travelers. Namely, if travelers choose to depart at e.g. time period 5 (while their preferred departure time is 10) then they will face high penalties for deviation from their preferred departure time. The same holds for deviating from preferred arrival time. That is the reason why the travel costs are (much) higher at the beginning and at the end of the studied time period (hence, time periods further away from PAT and PDT).

It should be also noted that the trip costs (between PDT and PAT time periods) are not smooth but show jumps. Because of simplicity, the trip costs are presented as one (resulting) curve (instead of separate cost curves for preferred departure and preferred arrival schedule delays). The jumps in cumulative cost curve are the result of forms of these two curves. Nevertheless, the dynamic equilibrium principle holds.

As can be seen from Figure 7.5a objective trip costs for no-toll case are lower in time periods from 8 to 16, which corresponds to the path flows in Figure 7.5b. Both classes of

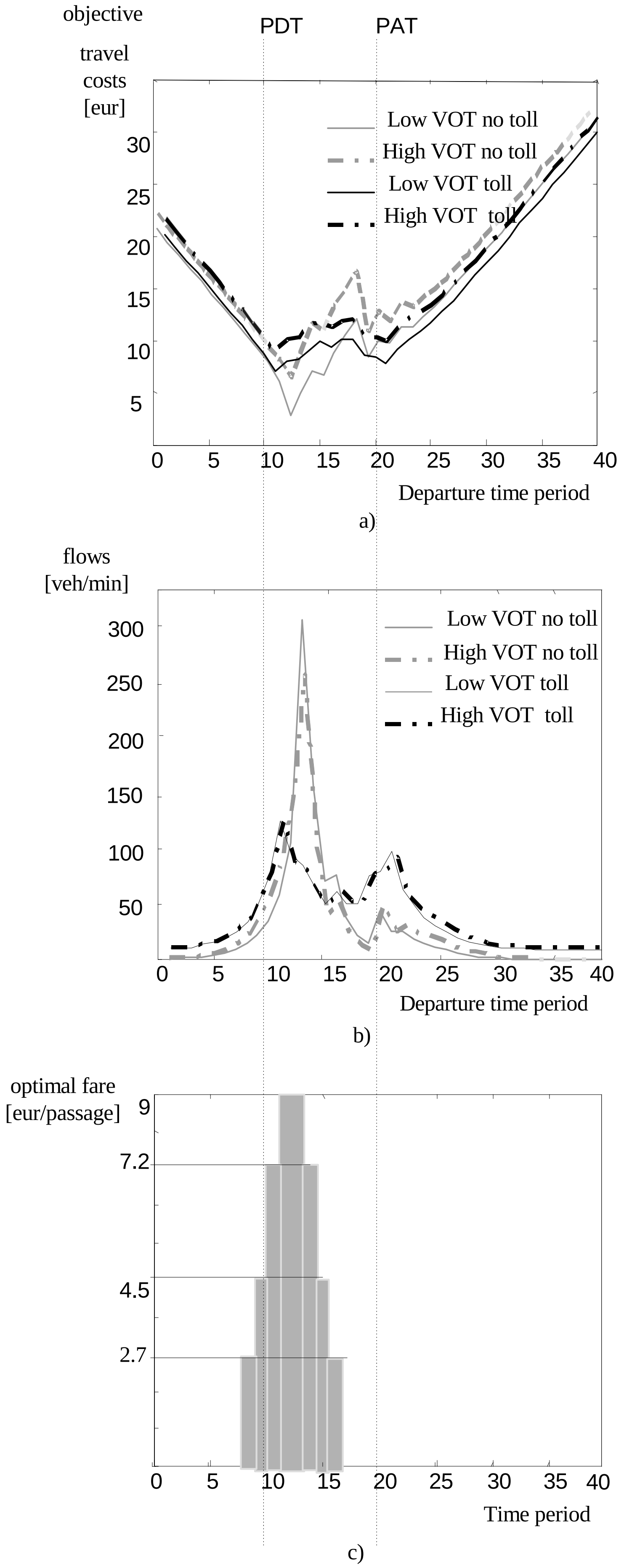

Figure 7.5: Results from experiment E1: Maximizing toll revenues with different VOT only:a) trip cost including tolls, b) path flows, c) resulting optimal toll pattern

Table 7.5: Number of paying and non-paying travelers by user class in experiment E1

	paying tolls	not paying tolls	$\sum$ total
group1 (low VOT travelers)	1000 (33%)	2300 (66%)	3300
group2 (high VOT travelers)	2000 (60%)	1300 (40%)	3300
$\sum$ total	3000 (45%)	3600 (55%)	6600

travelers appear to use the time periods from 8 to 25 because in these time periods they have lower travel costs.

If the tolling pattern (described in Figure 7.5c) is applied, then corresponding path flows on the corridor network are illustrated in Figure 7.5b, with black lines (solid for the travelers with low VOT and dashed for travelers with high VOT). Comparing the optimal trip costs and path flows for toll and non tolled situations (Figure 7.5a and Figure 7.5b), some findings appear. While in the case without tolls, even 300 vehicles per minute were in time period 14, in the case with tolls, only 66 vehicles per minute are present (strong peak reduction). In tolled periods, from 8 to 16, the lower number of travelers are present, while more outside of tolling periods. It should be noted that in the non-tolled case, as well as in the tolled case the proportions of different groups over time are almost constant. Sharp shifts occur in path flows and costs, for both no toll and toll case because of congestion in corresponding time periods. From this case study we can conclude that travel demand is shifted significantly towards the low-tolled time periods.

Figure 7.6 shows the revenue generation of the road authority for different values of tolls (from 0 to 50 eur/passage). If the optimal toll pattern (Figure 7.5c), resulting from the calculation for $\overline{\theta}$ is applied, then the optimal value of the objective function reached in this experiment is $Z_{revenue} = 156,7 \text{ x}10^2[eur]$. On the one hand, with zero tolls, no revenue is generated. On the other hand, if tolls imposed on the network are very high, then most of the travelers change their departure times in order to avoid to pay tolls (thus revenue is again close to zero). In our model we suppose that all travelers decide about their travel behavior according to utility maximization theory (see Chapter 6). Nevertheless, it may happen that some of the travelers still travel in tolled time periods despite extremely high tolls because of lower travel times and lower schedule delay costs.

In Table 7.5 different groups of travelers with regard to their VOT and paying tolls are given. While only 33 percent of travelers with low VOT paying tolls, even 60 percent of travelers with high VOT are willing to pay for a less congested trip.

From this experiment we learn about the impact of different toll values on revenue generation. Tolls may lead to a considerable shift in trip timing. While with lower tolls less revenue is generated, with high tolls less travelers are willing to pay, causing again low revenues. Thus, in this case study it is shown that with the optimal toll values the maximum of revenues can be reached. Furthermore, more insight is given on travel behavior of different user-classes if they differ in VOT. Depending on their value of time, it is shown that different groups of travelers behave differently depending on their travel preferences. Thus, more high VOT travelers are willing to pay tolls for an uncongested trip than the

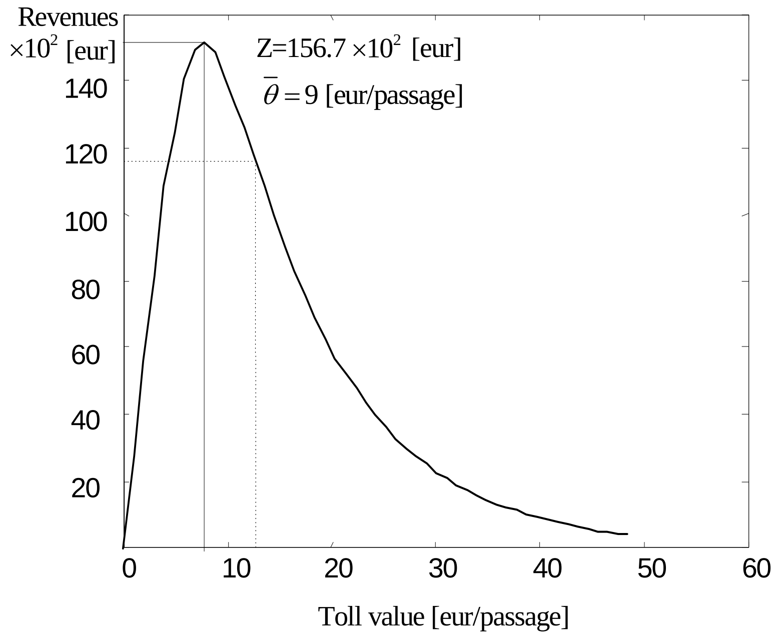

Figure 7.6: Experiment E1: Revenue outcomes by toll variation

travelers with low VOT.

Minimizing total travel time - Experiment $E2$ The other policy objective to be analyzed is to minimize the total travel time on the corridor network. It should be noted that this policy objective makes sense because total demand in these experiments is given, whereas to minimize total travel time in the case of elastic demand can result in reducing demand to zero.

The results of this case study (objective function and corresponding optimal toll pattern, respectively) are illustrated in Figure 7.7.

The optimal toll pattern (resulting from $\overline{\theta} = 11$ [eur/passage] in Figure 7.7a) is depicted in Figure 7.7b. Thus, in time periods $11, 12$, and 13 the highest fare value should be charged (11 eur/passage). In time periods 10 and 14 the fare is 8.8 [eur/passage] while in time periods 9 and 15 the fare is 5.5 [eur/passage]. The lowest tolls should be imposed in time periods 8 and 16 : 3.3 [eur/passage].

The impact of tolls are again strong peak reduction and demand redirection (towards less tolled time periods).

This tolling pattern generates the optimal value of the given objective function $Z_{time} =$ 254, 1 x $10^2 \Delta t$ [min]. It should be noted that due to tolls in these time periods, the demand is more evenly distributed also on other time periods.

From this experiment we learn about the impacts of tolls on policy objective of minimizing total travel time on the network. It seems possible to decrease total travel time on the

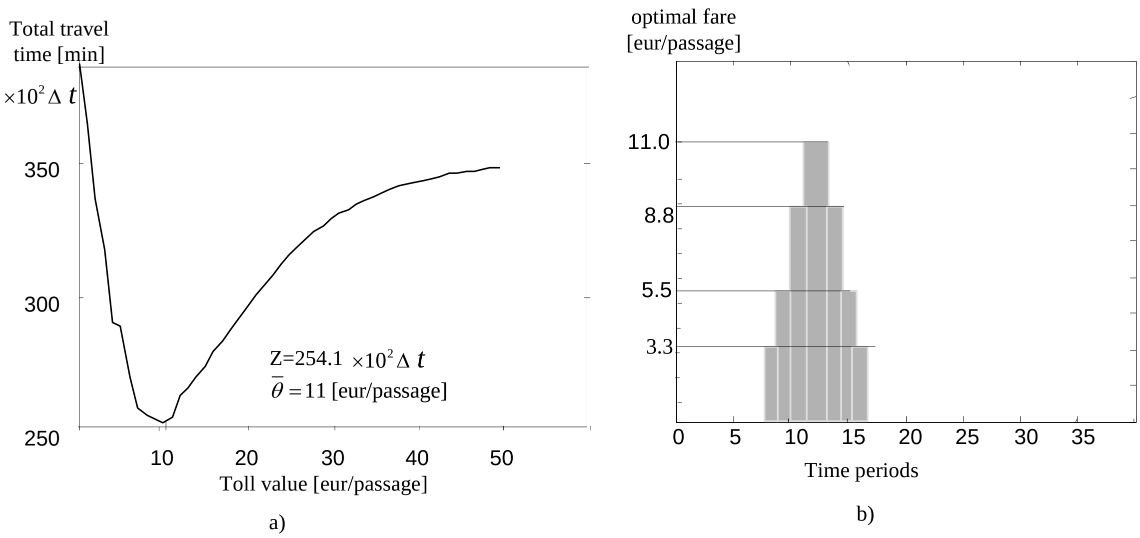

Figure 7.7: Results from experiment E2 with travelers with different VOT only: Total travel time minimization: a) value of objective function b) optimal toll value pattern

network applying tolls (from about 400 x $10^2 \Delta t$ to about 250 x $10^2 \Delta t$ [min]). Moreover, there is an optimal value of tolls that minimizes the objective function. Despite to the previous objective, the aim of road authority here is to redistribute travel demand more evenly over time and space.

Discussion of results of case studies $E1$ and $E2$ with travelers with different values of time. As can be seen from Figure 7.5*b* the travelers with high and low value of time have (almost) constant distribution per time periods. What is expected is that the travelers with low value of time will deviate more from tolled time periods than the travelers with high VOT (everything else than equal). It happens because the value of schedule delay of both groups of travelers is the same, hence there is no difference in schedule delay sensitivity between groups. However, more high VOT- drivers pay toll, implying that the difference in value of time plays an important role in travel behavior of different user classes. The motivation for the next set of experiments is to investigate how the difference in schedule delay sensitivity influence travel behavior of different classes of travelers. This is important because of gaining more insight into the role of schedule delay sensitivity of travelers in the DOTD problem.

In comparing both objective function cases E1 and E2, one may observe that not only the optimal toll values are similar but also that their corresponding objective function values are fairly close. In both cases, the objective is sensitive to small toll values.

Table 7.6: Input parameters for the corridor network, experiments E3 and E4: value of schedule delays for different groups

	parameter	*notation*	*value*	*unit*
1.	deviation from PDT of group 1	β_1	0.05	eur/min
2.	deviation from PDT of group 2	β_2	0.2	eur/min
3.	deviation from PAT of group 1	γ_1	0.08	eur/min
4.	deviation from PAT of group 2	γ_2	0.33	eur/min

7.4.4 Additional case studies ($E3$ and $E4$) with groups of travelers with different parameters for VOT and VOSD

Note that in Figure 7.5*b* the path flows (tolled case) for both groups of travelers are almost the same although the travelers with low VOT were expected to shift more to no-tolled time periods than the travelers with high VOT. In order to capture more realistic situations, it is necessary to include different parameters per group for schedule delays. With this we include an extra dimension of sensitivity of travelers, namely to time delays. We assume that travelers are willing to pay extra for being in time. Thus, in the following experiments $E3$ and $E4$ the deviation from preferred departure time of group 1 ($\beta_1 = 0.05$ [eur/min]) is valued lower than the deviation from PDT of group 2 ($\beta_2 = 0.2$ eur/min]). In other words, the travelers of group 1, with low value of schedule delay (VOSD) are less sensitive to departing earlier or arriving later than the travelers of group with high VOSD (group 2). In practice, if we consider only mandatory trips, it is reasonable that some travelers can use flexible working hours. The values of these parameters used in these experiments are outlined in Table 7.6.Due to these parameter values the no-toll cost and demand patterns shift a little bit.

Again we consider both policy objectives of maximizing total toll revenues ($E3$) as well as minimizing total travel time ($E4$).

Maximizing total toll revenue- Experiment $E3$ It should be noted that because of including of VOSD, the non-toll case looks different. Hence time periods in which tolls should be imposed are different than in case studies $E1$ and $E2$. The objective trip costs including zero-toll case and optimal tolls, the path flows as well as resulting optimal toll pattern (per time periods) are depicted in Figure 7.8.

In Figure 7.8*a* resulting objective trip costs of travelers with low VOT and VOSD (solid lines) as well as trip costs of travelers with high VOT and VOSD (dashed lines) are presented. At the same figure, the non-tolled objective trip costs are outlined (gray lines). Expectably, in both the tolled and non-tolled cases, high VOT+VOSD travelers have systematically higher objective travel costs for the same departure time.

In Figure 7.8*b* it can be seen that travelers with low VOT and VOSD will deviate more away from the tolled periods (and preferred departure and arrival periods) than travelers

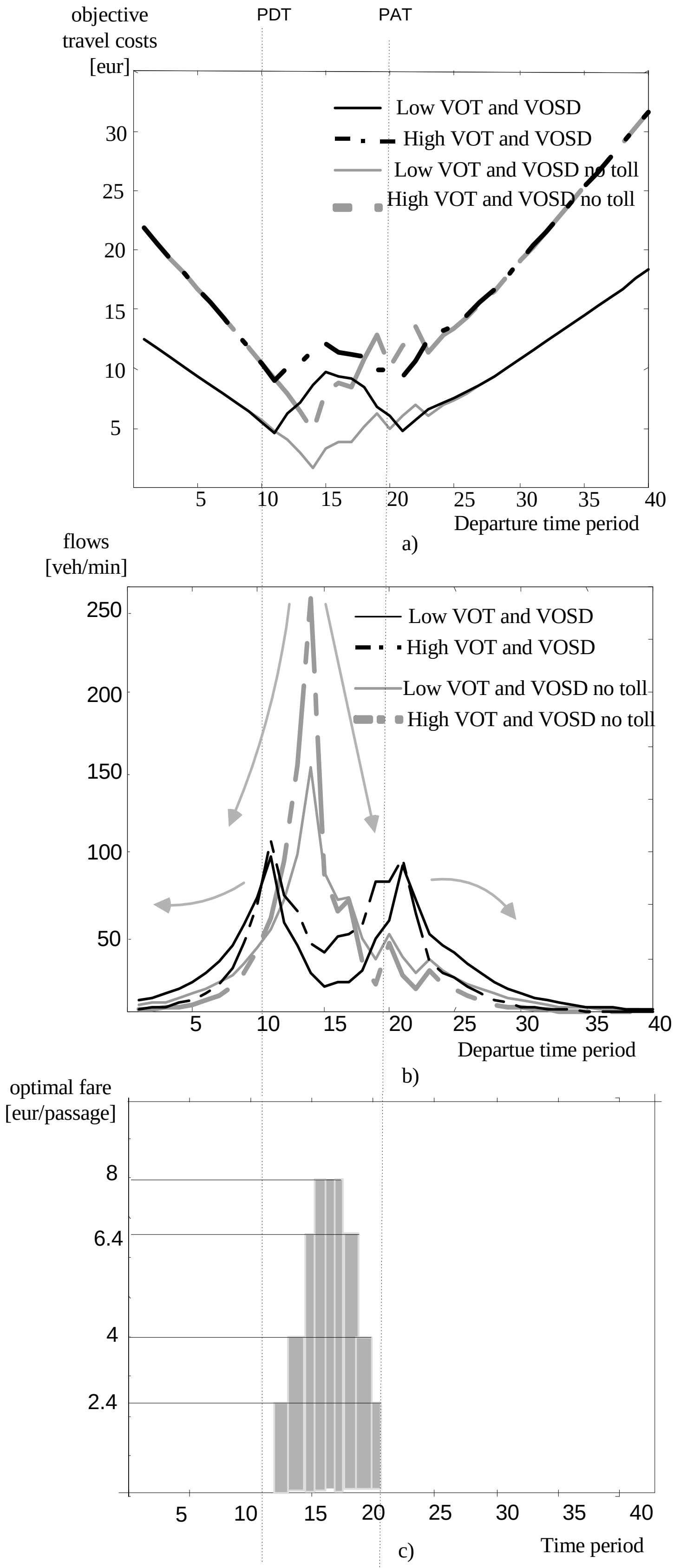

Figure 7.8: Results from experiment E3: Total revenue genaration for travelers with different VOT and schedule delays: a) path costs, b) path flows and c) resulting optimal toll values

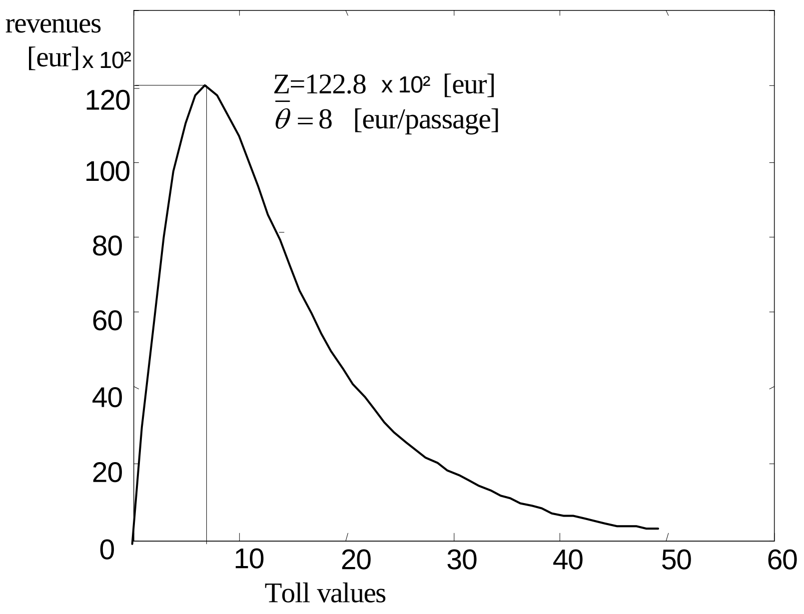

Figure 7.9: Experiment E3: Objective function of maximizing revenues all user classes

with high VOT and VOSD. Inside the tolled period and within PDT and PAT travelers appear who are *time sensitive* in contrast to *cost sensitive* travelers who are mostly outside of the tolled period and preferred departure and arrival periods.

If we compare the non-tolled (gray lines) and the tolled cases (black lines) in Figure 7.8*b* it can be seen that the travelers with low VOT and VOSD will deviate more from tolled periods and PDT and PAT than the travelers with only VOT differentiation. The explanation is that the travelers with low VOSD are willing to deviate more form preferred PDT and PAT than others.

The objective function of maximizing revenues is depicted in Figure 7.9. On the one hand, in the case where toll level is zero, there are clearly no revenues. On the other hand, for very high toll levels, all travelers will choose untolled time periods, resulting in zero revenues as well. As can be observed from Figure 7.9, the optimal toll value $\overline{\theta} = 8$ [eur/passages] results in the optimal toll time pattern (Figure 7.8*c*) yields the highest revenues $Z_{revenue} = 122, 8$ x 10^2 $[eur]$. Both these figures are lower than in the non-differentiated cases E1 and E2.

From this experiment we can learn about impacts of tolls on policy objective of maximizing total toll revenues if we consider differentiation between travelers with respect to schedule delay sensitivities. Temporal shifts are more pronounced especial a higher decline of demand in the tolled periods. It can be seen that with the optimal tolls the maximum of revenue generation is reached. The impact of different VOSD on travel behavior of different user classes is shown. The differentiation between t*ime sensitive* travelers (with high value of time and schedule delays) and *cost sensitive* travelers (those with low value of time and low schedule delays) is clearly made. Because of their differences in

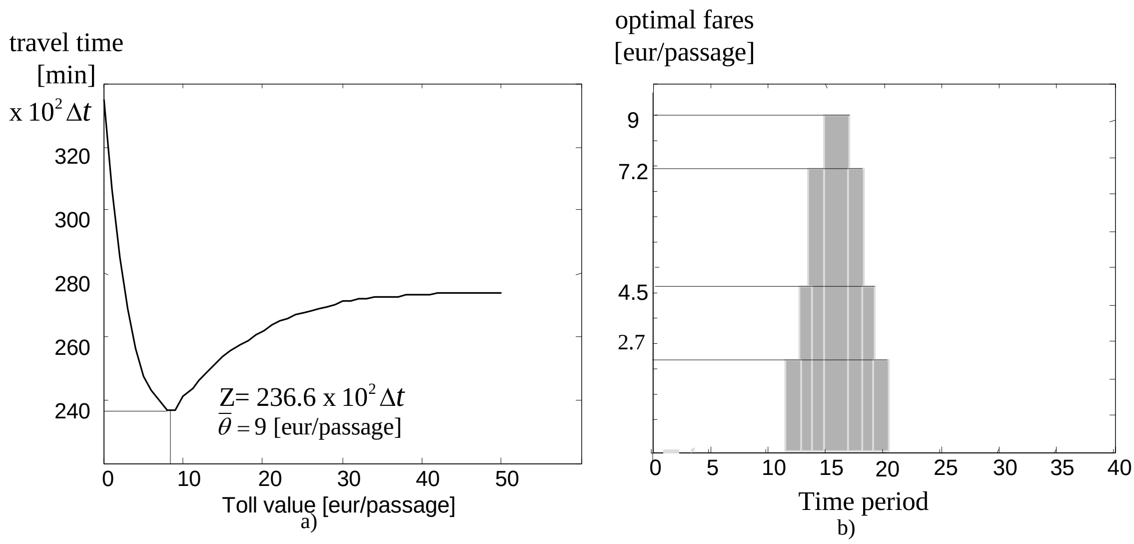

Figure 7.10: Results from experiment E4: Total travel time with travelers with different VOT+VOSD: a) objective function b) optimal temporal toll pattern

VOT and VOSD, travelers with low VOT and VOSD deviate more away from their PDT and PAT than the travelers with high VOT and VOSD.

Minimizing total travel time - Experiment $E4$ In Figure 7.10 the objective function values (7.10a), as well as resulting optimal toll values (Figure 7.10b) are presented.

As can be observed from Figure 7.10a, it appears possible to decrease the total travel time on the network by imposing a toll pattern on congested time periods due to better spreading demand over time. High toll levels push some travelers (mostly these with low value of time and low schedule delay) during the tolled period away from tolled periods and preferred PDT and PAT. If the optimal toll pattern (resulting from $\overline{\theta} = 11$ [eur/passage] in Figure 7.10b) is applied, the optimal value of the objective function is $Z_{time} = 236, 6$ x $10^2 \Delta t$ [min].

Discussion of the results of experiments $E3$ **and** $E4$ **with different VOSD** From this experiment we can see impacts of tolls on policy objective function of minimizing total travel time where different groups with respect to VOSD are included. It is shown that the total travel time on the network can be minimized (from about 340 x $10^2 \Delta t$ to about 240 x $10^2 \Delta t$ [min]) if the optimal toll pattern (7.10b) is applied.

From the results presented in experiments $E3$ and $E4$ the effects of different VOSD user classes on travel patterns can be observed. Because of differences in VOT and VOSD, time sensitive travelers (high VOT and VOSD) are willing to pay for an less congested trip than cost sensitive travelers (these with low VOT and VOSD). Namely, the travelers with lower VOSD accept to travel outside of their preferred departure and arrival time

Table 7.7: A comparison of the corridor experiments with respect to optimal tolls and resulting values of objective functions

		VOT only ($E1$, $E2$)	*VOT +VOSD* ($E3$, $E4$)
	$\overline{\theta}_a$ [$eur/passage$]	9	8
max REV	Z_{rev} [eur]	156, 7x10^2	122.8x10^2
	num of travelers paying	3220	2494
	$\overline{\theta}_a$ [$eur/passage$]	11	9
min TT	Z_{time} [$hour$]	254.1x10^2 Δt	236.6x10^2 Δt
	num of travelers paying	3716	2220

because their penalties for departing earlier and arriving later are lower than those of travelers with higher values of schedule delays.

In comparizing both objective function cases E3 and E4, one may observe that not only optimal toll values are similar but also that their corresponding objective function values are fairly close. Both objectives are sensitive to small toll values.

7.4.5 Discussion of corridor experiments ($E1$ - $E4$)

A comparison of corridor experiments conducted in this section is given in Table 7.7. For every experiment, the resulting optimal maximum toll value $\overline{\theta}$, the optimal objective function value Z, and the total number of travelers (of both groups) in tolled time periods are presented. With inclusion of VOSD, the optimal tolls to be charged are lower than in the case with VOT only (for both objective functions). While in the case of revenue generation it results in lower total revenue generation, in the case of travel time objective with lower optimal tolls the lower travel time is reached. It can be explained by the aim of these objectives. While in the case of revenue generation, the aim is to generate as much as possible revenue (hence higher tolls) the aim of the other policy objective is to spread the travel demand over routes and time period more evenly.

From this table, we can derive some conclusions. If only VOT is taken into account, then the maximum revenues are 156, 7 x10^2 eur. But, if we include additional distinction in VOSD, then more travelers (those with low VOSD) accept to travel outside of peak periods. Thus, the earned revenues are lower (122.8x10^2 [eur]). As a result less travelers are willing to pay if they should deviate from their preferred PDT and PAT.

However, for the policy objective of minimizing total travel time, this extension with VOSD is beneficial, because of more evenly spread travel demand over time periods (total travel time is reduced from 254, 4 x10^2 Δt to 236.6x10^2 Δt) which is supported with fewer travelers paying toll.

An additional analysis of travel behavior of different groups of travelers per experiment is given in Table 7.8. From the analysis it can be seen how many travelers are willing

Table 7.8: An analysis of participation of different groups in tolled periods

objective		*VOT only* ($E1$, $E2$)	*VOT + VOSD* ($E3$, $E4$)
	low group	*1534*	*910*
max REV	high group	*1686*	*1584*
	total num of travelers	**3220**	**2494**
	low	*1843*	*793*
min TT	high	*1873*	*1427*
	total num of travelers	**3716**	**2220**

to travel in *tolled periods* and from which groups, as well as the total number of travelers. Note that the total demand is 6600 travelers, where in both of groups 3300 travelers are assigned. While in the case with the VOT distinction only, the distribution among groups is almost uniform, in the case with the additional schedule delay distinction, the distribution between different groups is more sharp.

Thus, in both cases (revenue generation and minimization of travel time) less travelers are willing to pay. That happens especially with the travelers with low VOT and VOSD who deviate much more from tolled periods than travelers with high VOT and VOSD. In the case of revenue generation from 1534 to 910 and in the case of minimization of travel time from 1843 to 793. As expected, the number of high VOT and VOSD travelers is not changed so much (in the case of revenue generation from 1686 to 1584 and in the case of travel time form 1873 to 1427) shown that they are willing to pay more for an uncongsted trip. These findings are in line with the literature, see e.g. Ubbels (2006), Verhoef et al. (2004).

It can be concluded that the schedule delay sensitivity will play an important role in the optimal toll design problem. In other words, an additional difference in schedule delay valuation between travelers, makes a significantly improved and more realistic modeling of travel behavior.

7.5 Case studies with dual route network ($E5$-$E10$)

In this section, a series of experiments on a dual route network are conducted. Now route choice is also a travel option. The aim of these experiments is to demonstrate various temporal tolling patterns (uniform, (quasi)uniform and variable) in the DOTD problem (see Table 7.1).

7.5.1 Network description

The network used in these examples consists of just a single-OD pair connected by two non-overlapping paths where only link 2 is tolled (see Figure 7.11). Since there is only one OD pair, we will ignore the OD subindices (r, s) in variables.

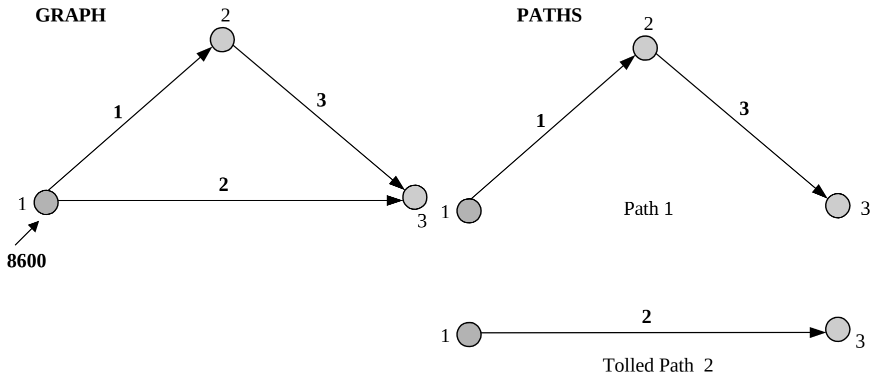

Figure 7.11: Description of dual-route network in experiments E5-E10

Table 7.9: Link travel time function parameters for the dual network

	parameter	link 1	link 2	link 3
1.	free flow time (τ_a^0) [min]	$\tau_1^0 = 10.5$	$\tau_2^0 = 9.0$	$\tau_3^0 = 10.5$
2.	impact on volume (b_a)[min/veh]	$b_1 = 0$	$b_2 = 0.005$	$b_3 = 0$
3.	power of the volume (c_a)	$c_1 = 1$	$c_2 = 1$	$c_3 = 1$

7.5.2 Link travel time functions

We assume the same travel time function as (7.5) although with specific parameter values. The parameters of link travel time functions of this dual network are given in Table 7.9.

At link level, we assume that route 1 with a free flow travel time of 21 minutes is longer than route 2 (9.0 minutes) by setting the free flow travel times in Table 7.9. Furthermore, it is assumed that the first route is of higher capacity and will never have congestion, hence $b_1 = b_3 = 0$, while congestion is possible for link (route) 2 for which we set $b_2 = 0.005$ [min/veh] .

Travel demand description and input parameters

Two user classes with different values of time (VOT) are distinguished. The total travel demand for departure period $K = \{1, ..., 20\}$ from node 1 to node 3 is $D_{total}^{rs} = 8600$ vehicles, of which 50% high VOT travelers and 50% low VOT travelers. The parameter values used at path level are shown in Table 7.10.

The parameters of the dynamic route cost function, Equation (6.3) are given in Table 7.10 as well as the parameter μ of the joint route-departure logit choice model (Chapter 6, Equation (6.7)).

Table 7.10: Parameters for the dual traffic network: demand side

	parameter	*notation*	*value*	*unit*
1.	preferred departure time	PDT	10	Δ
2.	preferred arrival time	PAT	15	Δ
3.	departure time intervals	K	20	-
4.	deviation from PDT (for both groups)	β	0.08	eur/min
5.	deviation from PAT (for both groups)	γ	0.33	eur/min
6.	VOT group 1	α_1	0.08	eur/min
7.	VOT group 2	α_2	0.25	eur/min
8.	scale parameter (logit model)	μ	0.80	-

7.5.3 Zero-toll case

For the case without tolls, the dynamic route costs and flows are depicted in Figure 7.12. The flows appear almost evenly spread between the two routes. The departure time profiles indicate that the travelers that use longer route 1 will depart earlier in order to arrive as close as possible to their preferred arrival time.

7.5.4 Toll pattern

In this case study three different tolling regimes will be used, namely uniform, (quasi) uniform and variable tolling scheme (see Section 7.2). Note that in this case study with only a single link (link 2) tolled, determining the optimal toll for each tolling regime (even for the variable tolling scheme) only requires to find a single optimal toll level, $\overline{\theta}_2$.The uniform tolling scheme only has a single parameter. This toll can only be escaped switching route. For the (quasi)uniform tolls we assume that a toll will be levied in peak time periods only. According to Figure 7.12, this tolling period is $T^{'} = \{8, 9, 10, 11, 12\}$. In the variable tolling scheme only time periods 9, 10, 11 will be tolled with fixed proportions 0.6, 1.0 and 0.6. In the cases of (quasi)uniform and variable tolls also departure time shifts are relevant to escape from tolls .

$$\phi_2(t) = \left\{ \begin{array}{ll} 1.0, & \text{if } t = 10; \\ 0.6, & \text{if } t = 9, 11; \\ 0, & \text{otherwise.} \end{array} \right\} \tag{7.9}$$

7.5.5 Results with tolls

Objective: Maximizing toll revenues (*E*5, *E*6, *E*7) with different tolling schemes

Assume that the road authority aims to maximize total revenues, as formulated in Chapter 5, by selecting the best tolling scheme and the best toll level. The three different tolling

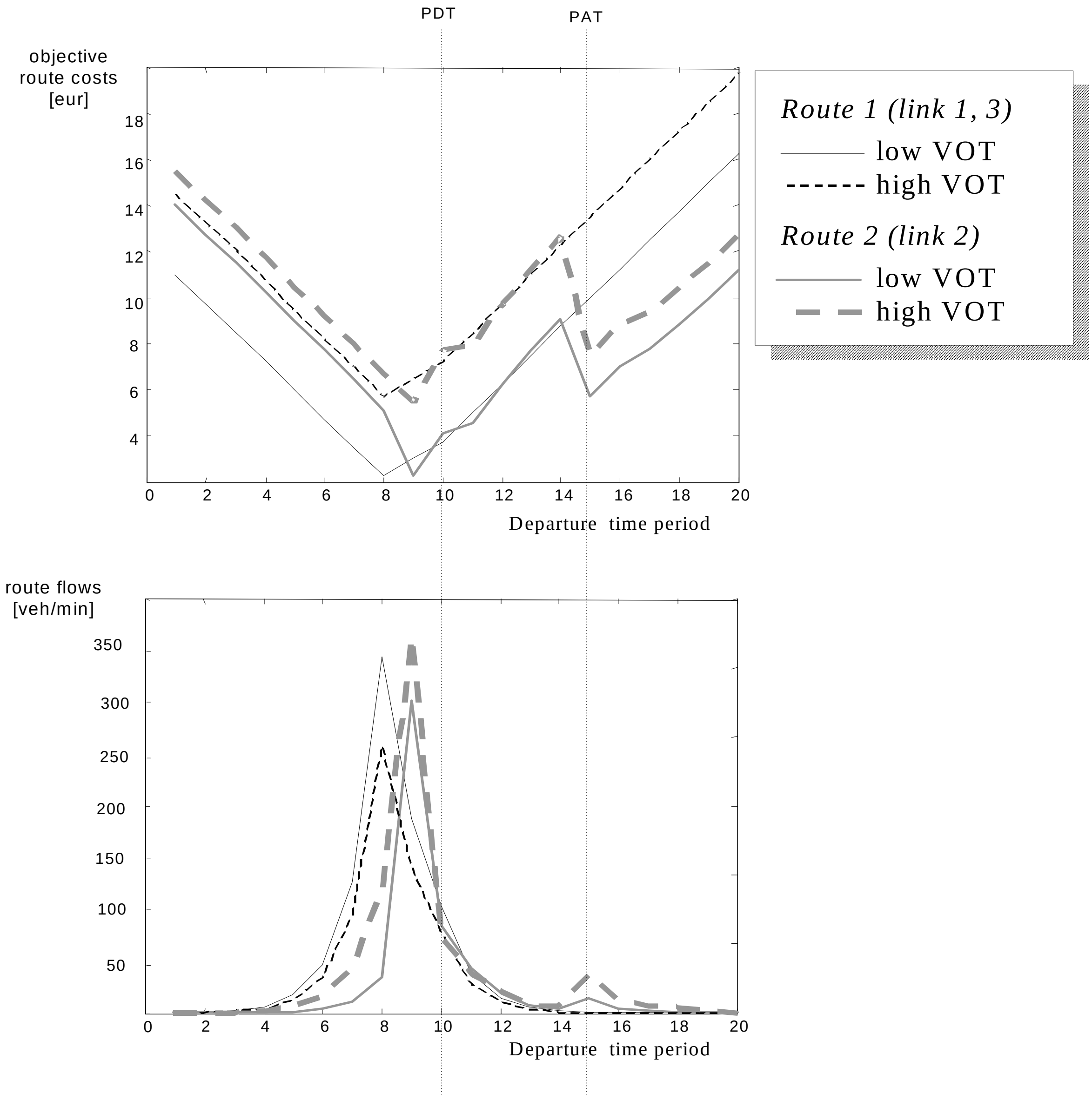

Figure 7.12: Dual network: dynamic route flows and costs in the case of zero tolls

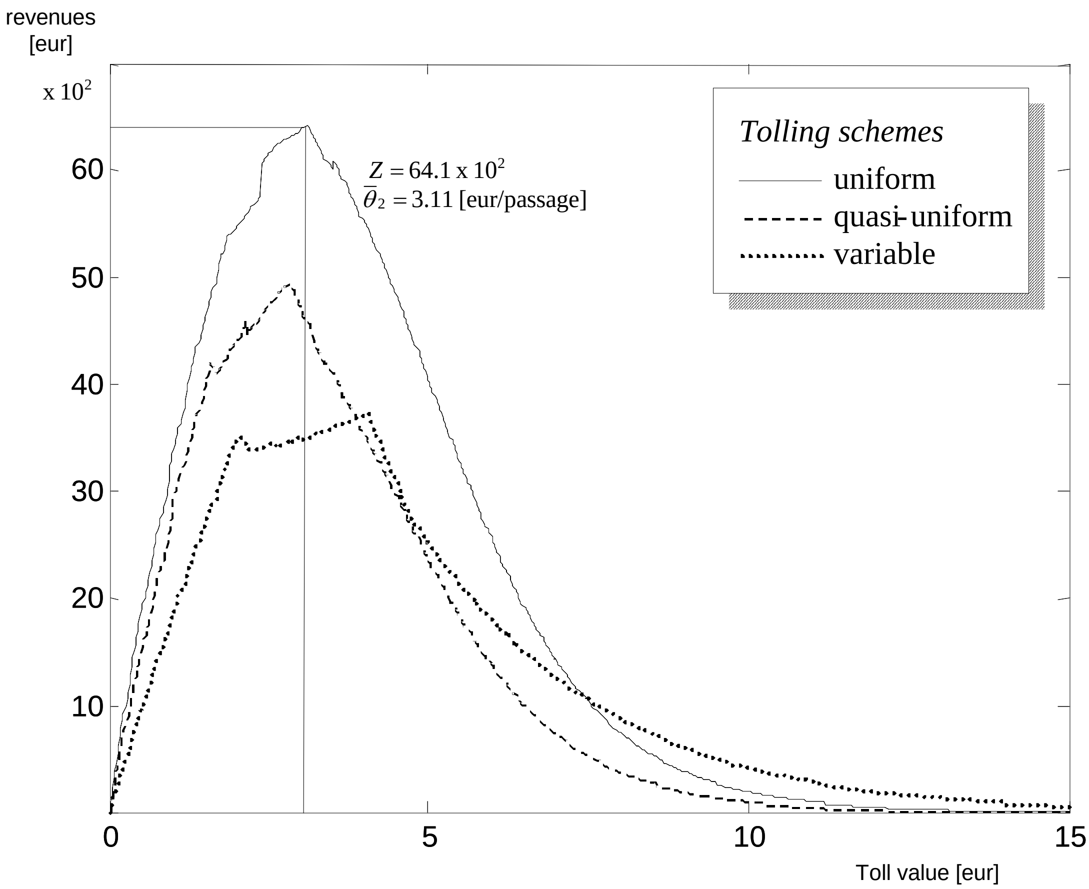

Figure 7.13: Results of experiments E5, E6 and E7: Total toll revenues for different tolling schemes and toll levels

regimes (see Section 7.2) will be considered in this case study. For each tolling regime and each toll level, the dynamic traffic assignment (DTA) will be solved. In Figure 7.13, the total revenues are plotted for each tolling scheme for all $0 \leq \overline{\theta}_2 \leq 15$ [eur].Although not shown here, in all cases the DTA model converged. In the case the toll level is zero, there are clearly no revenues. For very high toll levels, all travelers will choose to travel on the untolled route, resulting in zero revenues as well. As can be observed from Figure 7.13 uniform tolling with $\overline{\theta}_2 = 3.11$ yields the highest revenues. The variable tolling scheme is not able to provide high revenues due to a limited number of tolled time intervals.

The route flows and costs are depicted in Figure 7.14, together with the optimal toll levels for the objective of maximizing toll revenues. Compared with the case of no tolls in Figure 7.12, it can be seen that travelers shift towards (non-congested and untolled) route 1 and also shift their departure times (mostly later). Furthermore, it can be observed that there are many more travelers with high VOT on tolled route 2 then travelers with a low VOT. This is to be expected as travelers with high VOT care less about toll costs and more about short trip time.

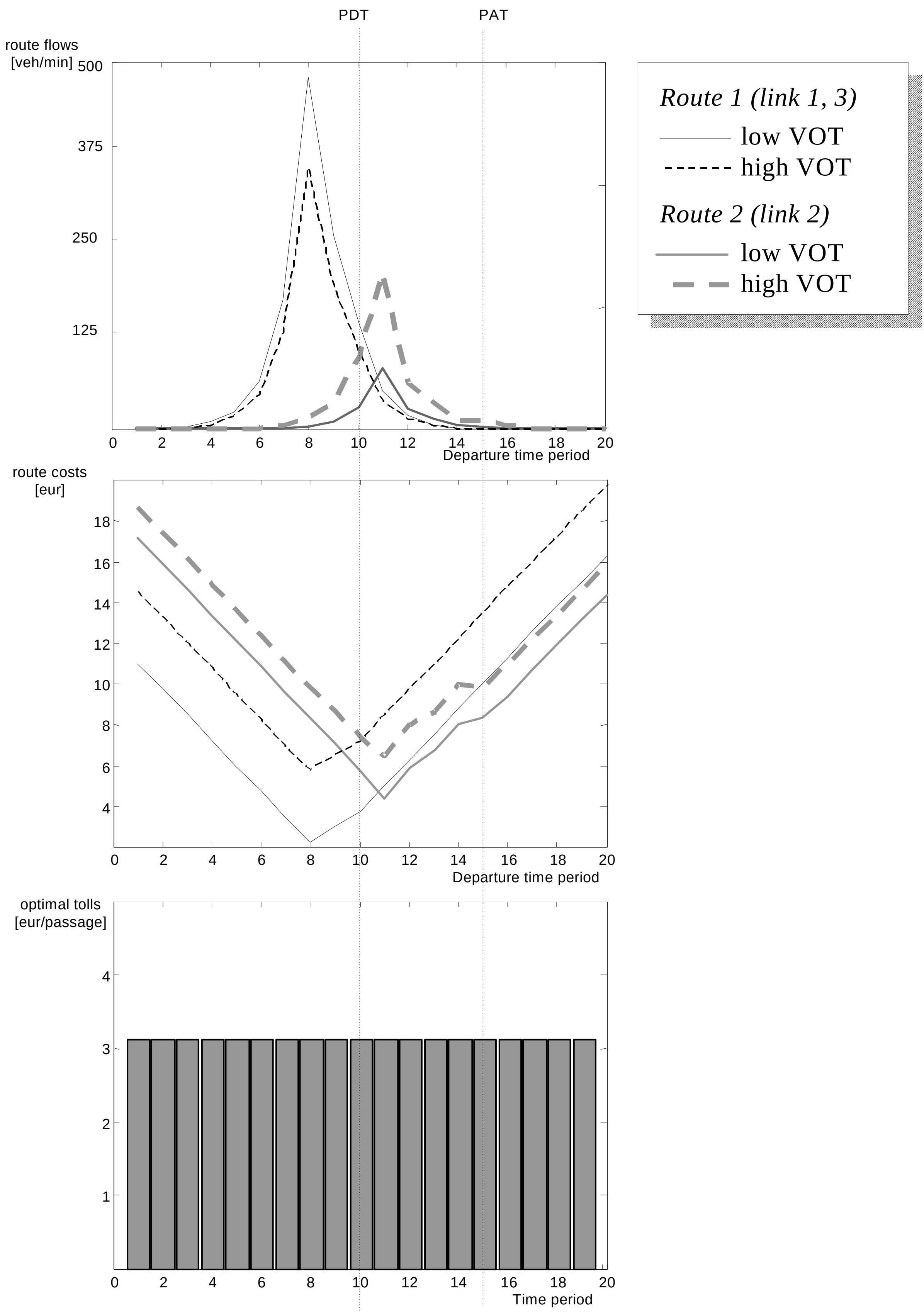

Figure 7.14: Route costs, flows and optimal uniform toll when maximizing revenues

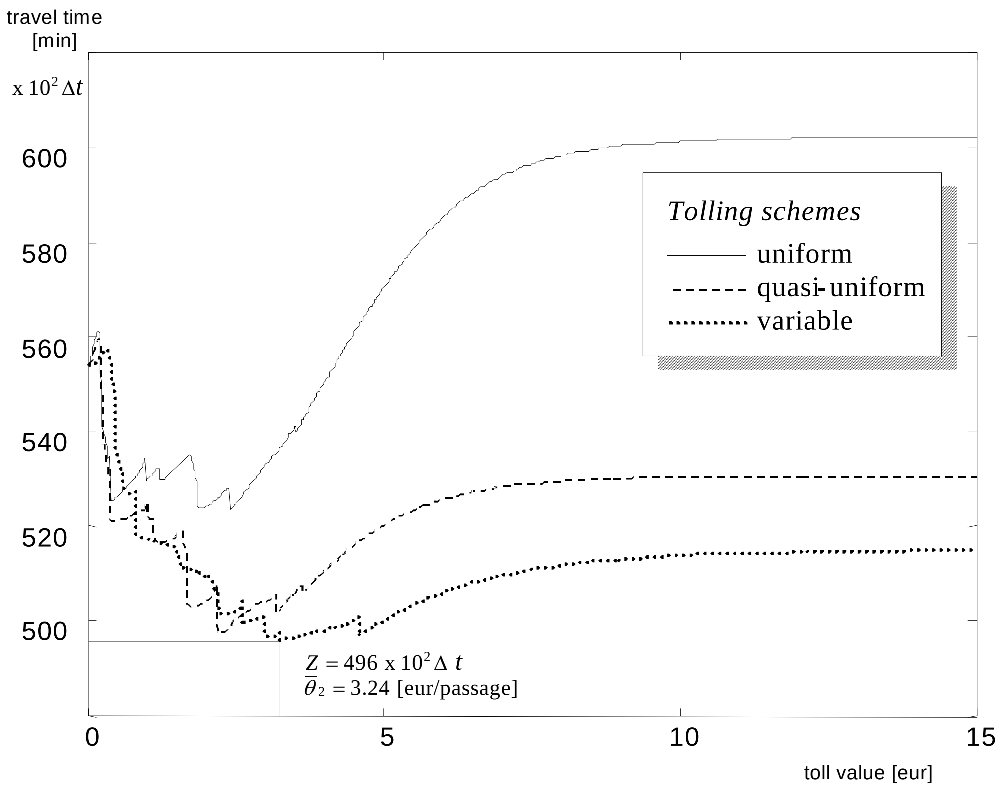

Figure 7.15: Results of experiments E8, E9 and E10: Total travel time for different tolling schemes and toll levels

Objective: Total travel time minimization ($E8$, $E9$, $E10$)

In these case studies the road authority aims at minimizing total travel time on the network by selecting the best tolling scheme and the best toll levels. Figure 7.15 depicts the total travel times for different tolling schemes and toll levels.

As can be observed from Figure 7.15, it seems possible to decrease total travel time on the network by imposing a toll on congested route 2. High toll levels will push more travelers during the toll period away from route 2 to the longer route 1, yielding higher total travel times again. Variable tolling with $\overline{\theta}_2 = 3.24$ (yielding $\theta_2(10) = 3.24$ and $\theta_2(9) = \theta_2(11) = 1.99$) results in the lowest total travel time. The objective function looks somewhat irregular. However, this can be explained by the rounding off of the link travel times in flow propagation equation (time discretization).

The route costs and flows are depicted in Figure 7.16, together with optimal toll levels for the objective of minimizing total travel time on dual network. Compared with Figure 7.14, it can be clearly seen that there are more departure time changes due to the fact that only the peak period is tolled, leading to a better spread of traffic over space and time and therefore lower total travel time.

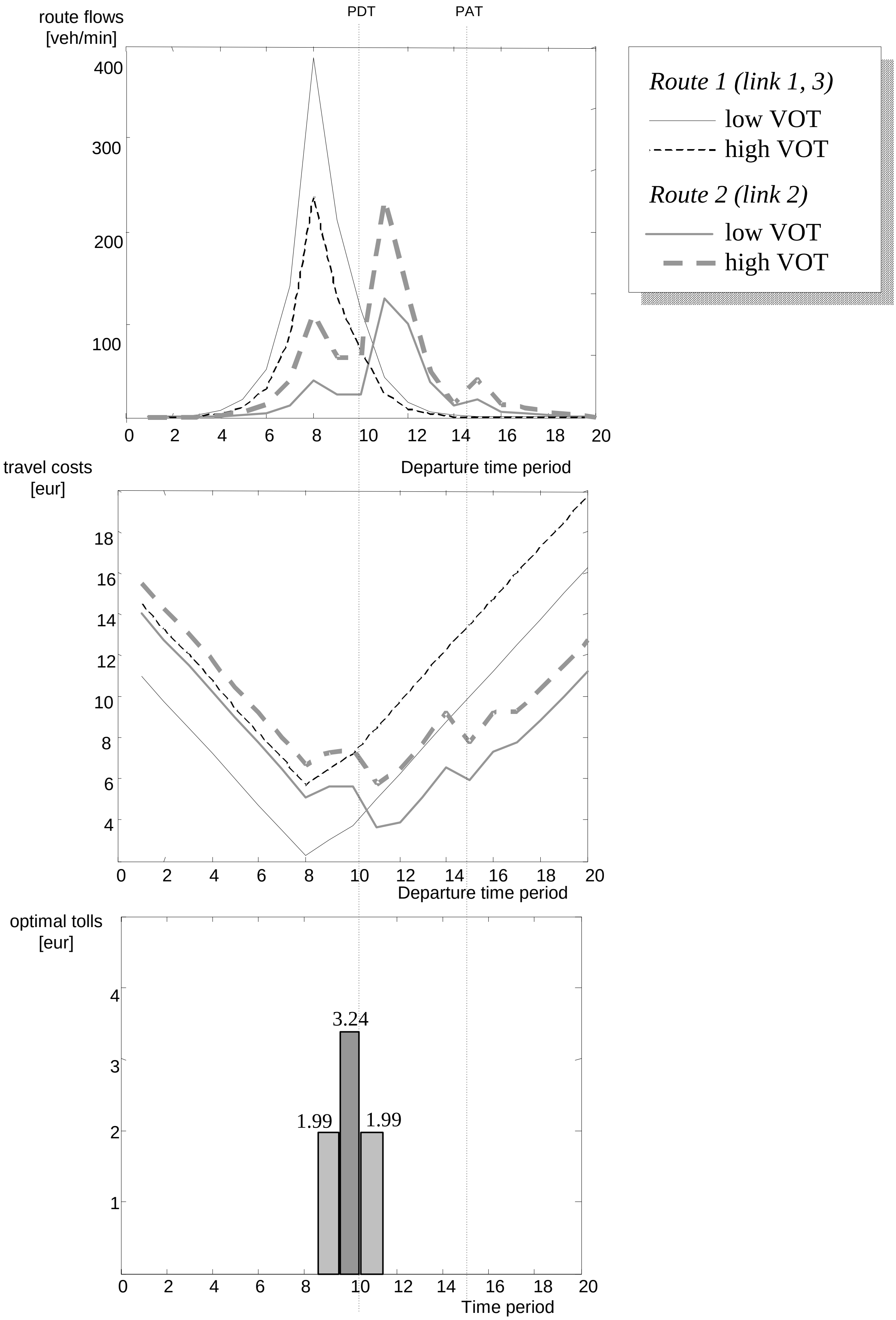

Figure 7.16: Route flows, costs and optimal tolls for minimizing total travel time

Table 7.11: Comparison of total toll revenues and travel times for different tolling schemes

Objective: maximize total toll revenue			
tolling scheme	*Optimal tolls*	*total revenues*	*total travel time*
uniform	3.11	$64.15\text{x}10^2$	$534.48\text{x}10^2\,\Delta t$
(quasi)uniform	2.82	$49.50\text{x}10^2$	$503.42\text{x}10^2\,\Delta t$
variable	4.04	$37.13\text{x}10^2$	$498.26\text{x}10^2\,\Delta t$
Objective: minimize total travel time			
tolling scheme	*Optimal tolls*	*total revenues*	*total travel time*
uniform	2.41	$60.81\text{x}10^2$	$523.56\text{x}10^2\,\Delta t$
(quasi)uniform	2.27	$45.56\text{x}10^2$	$497.91\text{x}10^2\,\Delta t$
variable	3.24	$35.25\text{x}10^2$	$496.02\text{x}10^2\,\Delta t$

7.5.6 Discussion of experiments $E5 - E10$

Results of both policy objectives (maximizing total toll revenues and minimizing total travel time) with different tolling schemes (uniform, (quasi)uniform and variable) are outlined in Table 7.11.

The results show that in the case of maximizing toll revenues the best tolling scheme is uniform with toll level $\overline{\theta}_2 = 3.11$. People cannot escape by switching travel timings. However, this toll value will yield a high total travel time ($534.48\text{x}10^2\,\Delta t$ [min]). On the other hand, in the case of minimizing total travel time, the variable tolling scheme with $\overline{\theta}_2 = 3.24$ performs best. However this toll will yield a low total toll revenue ($35.25\text{x}10^2$ [eur]). In other words, maximizing total toll revenue and minimizing total travel time are opposite objectives. This can be explained as follows. In maximizing toll revenues, the road authority would like to have as many as possible travelers on the tolled route, hence trying to push as few as possible travelers away from the tolled alternative by imposing toll. In contrast, when minimizing total travel time, the road authority would like to spread the traffic as much as possible in time and space, hence trying to influence as many travelers as possible to choose other departure times and routes. Using a uniform tolling scheme, travelers are not changing their departure times, making it suitable for maximizing revenues, while in the variable tolling scheme other departure times are good alternatives, making it suitable for minimizing travel time. In any case, depending of the objective of the road authority, there are different tolling schemes with different toll levels.

7.6 CASE Study 3: Case studies with a multiple OD-pair network ($E11$, $E12$)

In this section, a few experiments on a simple multiple OD-pair network are described. The purpose of this section is to apply the proposed model to solve the DOTD problem on

Table 7.12: Parameters of the link travel time functions for Chen nework

	free flow time (τ_a^0)	impact on x_a (b_a)	power of x_a (c_a)
link	[min]	[min/veh^2]	
1.	$\tau_1^0 = 9.0$	$b_1 = 0$	$c_1 = 1$
2.	$\tau_2^0 = 12.0$	$b_2 = 0.25$	$c_2 = 1$
3.	$\tau_3^0 = 9.0$	$b_3 = 0$	$c_3 = 1$
4.	$\tau_4^0 = 9.0$	$b_4 = 0$	$c_4 = 1$
5.	$\tau_5^0 = 12.0$	$b_5 = 0.25$	$c_5 = 1$
6.	$\tau_6^0 = 9.0$	$b_6 = 0$	$c_6 = 1$

a multiple OD-pair network, by setting toll values on multiple links in order to optimize given policy objectives.

7.6.1 Network description

The so-called 'Chen network' (Chen (1999)) used in these examples consists of two OD-pairs. The graph and constitution of paths of the Chen network are depicted in Figure 7.17. The Chen network was chosen for these experiments because of the existence of overlapping routes, to simulate the situation with 'shorter' but congested routes and 'longer' (and usually less congested routes). In this kind of networks, it will be interesting to investigate the situation where these 'congested' routes are tolled.

The first OD pair (OD-1) in Figure 7.17 connects node 1 to node 5, while the second OD pair (OD-2) connects node 3 to node 5. Clearly, there are four (partly) overlapping paths for OD-1, in Figure 7.17 noted as path 1, path 2, path 3 and path 4. Path 1 consists of links 1, 3, 4 and 6; path 2 consists of links 1, 3 and 5; path 3 consists of link 2, 4 and 6 while path 4 consists of links 2 and 5. There are two non-overlapping paths for OD-2: path 5 (consists of links 4 and 6) and path 6 (only one link 5).

In the real situation, this network can represent the highway network connecting main Dutch cities (The Hague and Rotterdam) via smaller cities (Delft, Zoetermeer and Gouda). The OD flows in this network interact at links 4, 5 and 6.

7.6.2 Link travel time functions

Link travel time functions in these case studies ($E11$ and $E12$) have again the form as in Equation (7.5).

Table 7.12 lists the values of the link travel time parameters (τ_a^0, b_a, c_a) that are used in this experiment.

We assume that free flow travel times for links 1, 3, 4 and 6 are 9 minutes while free flow travel times for links 2 and 5 are 12 minutes. Furthermore, we assume that links 1, 3, 4

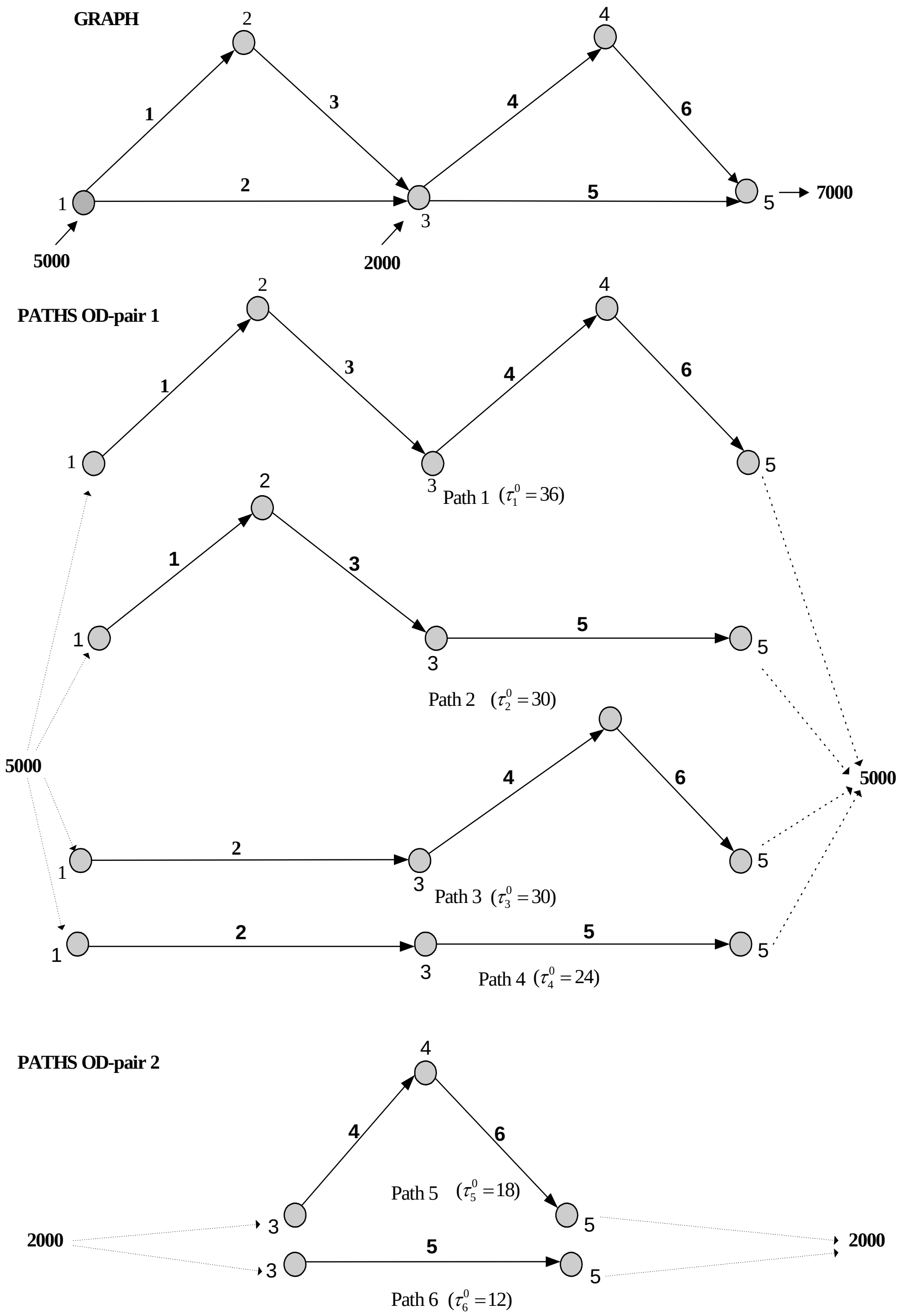

Figure 7.17: Description and path constitution for the multiple OD-pair network used in experiments E11 and E12

Table 7.13: Parameters for Chen network: demand side

	parameter	*notation*	*value*	*unit*
1.	preferred departure time OD 1	PDT^1	10	Δ
2.	preferred arrival time OD 1	PAT^1	20	Δ
3.	preferred departure time OD2	PDT^2	15	Δ
4.	preferred arrival time OD2	PAT^2	20	Δ
5.	departure time intervals	K	30	-
6.	deviation from PDT (for both groups)	β	0.08	eur/min
7.	deviation from PAT (for both groups)	γ	0.33	eur/min
8.	VOT group 1	α_1	0.08	eur/min
9.	VOT group 2	α_2	0.25	eur/min
10.	scale parameter (path-size model)	η	0.50	-

and 6 never have congestion due to sufficient capacity while congestion is possible on links 2 and 5, because of limited capacity, hence we set parameters b on links 2 and 5 to 0.25 [min/veh].

Thus, route 1 (consisting of links 1, 3, 4, 6) has always travel time of 36 minutes irrespective of loads, while route 4 (consisting of only links 2 and 5) has minimum travel time of 24 minutes (but this route can be congested). Clearly, the idea is to set tolls on these congested links 2 and 5 of the Chen network in order to determine optimal toll values to satisfy given policy objectives.

7.6.3 Travel demand description and input parameters

Two user classes with different values of time (VOT) are distinguished in this case study. The total travel demand for the whole period $K = \{1, ..., 30\}$ is $D_{total} = 7000$, of which $D^{15}_{total} = 5000$ are travelers from node 1 to node 5, while $D^{35}_{total} = 2000$ are travelers from node 3 to node 5. Furthermore, for each OD pair, we assume that 50% of travelers have high VOT and the other 50% low VOT.

The parameter values, used on path level, are shown in Table 7.13.

In order to capture the phenomenon of overlapping routes, the path-size model (introduced in Chapter 6) for all overlapping paths is used where path-size factors are $PS = 0.5$. The path-size factors for all 6 routes are equal in this experimental set-up.

7.6.4 Zero toll case

For the non-toll case, link travel times are depicted in Figure 7.18. On links 2 and 5 congestion appears. If only one of these two links would be tolled, the congestion on the other link would at least be unchanged or most probably increase. Because at both links different OD-trips appear, it makes sense to toll both links. Thus, the proposed time

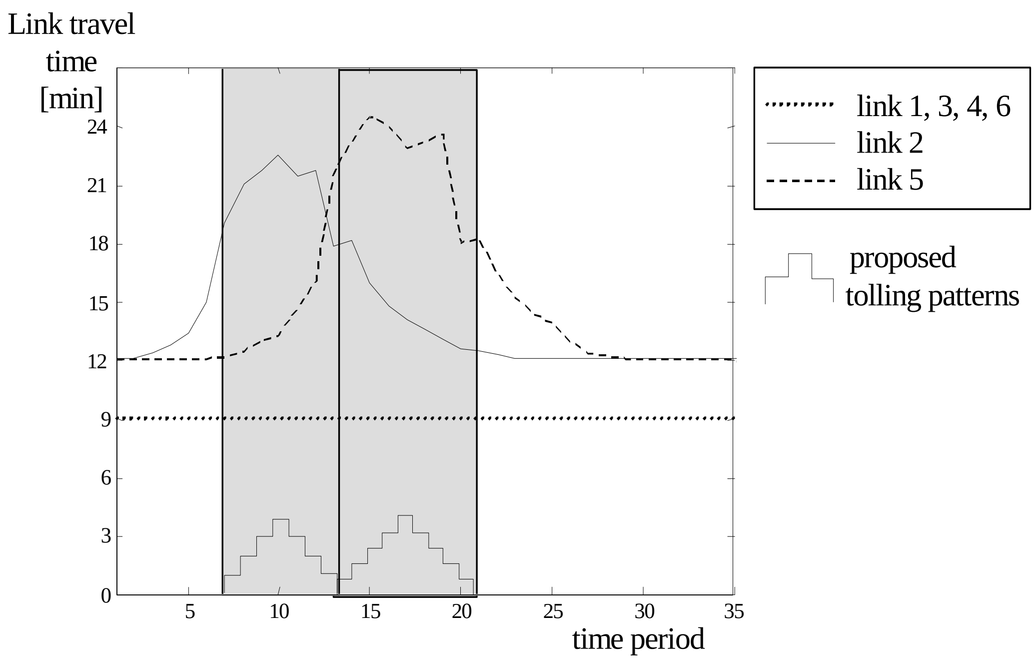

Figure 7.18: Link travel times on Chen network: zero toll case

periods to set tolls on links 2 and 5 will correspond to the congested time periods on links 2 and 5 in the zero toll case (see Figure 7.18).

For the non-tolled case, entering route flow rates are depicted on Figure 7.19. To prevent complexity, route flow rates for travelers with high VOT and low VOT are shown together. From Figure 7.19 we can see how many travelers use which route (in this untolled case) in order to compare these route flows with the tolled case. It is expected that travelers shift to untolled routes and time periods.

7.6.5 Toll pattern

Only a variable tolling scheme will be adopted. We aim to set toll values on link 2 (part of routes 3 and 4) and link 5 (part of routes 2, 4 and 6). Note that route 4 consists of two links which are both tolled. According to Figures 7.18 and 7.19, best time periods of tolling for link 2 seem $T_2^{'} = \{7, 8, 9, 10, 11, 12, 13\}$, while the best tolling pattern for link 5 is different, namely $T_5^{'} = \{13, 14, 15, 16, 17, 18, 19, 20, 21\}$. Similarly to previous case studies, only selected time periods will be tolled with prespecified (fixed) proportions. On link 2 the following proportions will be applied:

$$\phi_2(t) = \left\{ \begin{array}{ll} 1.0, & \text{if } t = 10; \\ 0.75, & \text{if } t = 9, 11; \\ 0.50, & \text{if } t = 8, 12; \\ 0.25, & \text{if } t = 7, 13; \\ 0, & \text{otherwise.} \end{array} \right\} \tag{7.10}$$

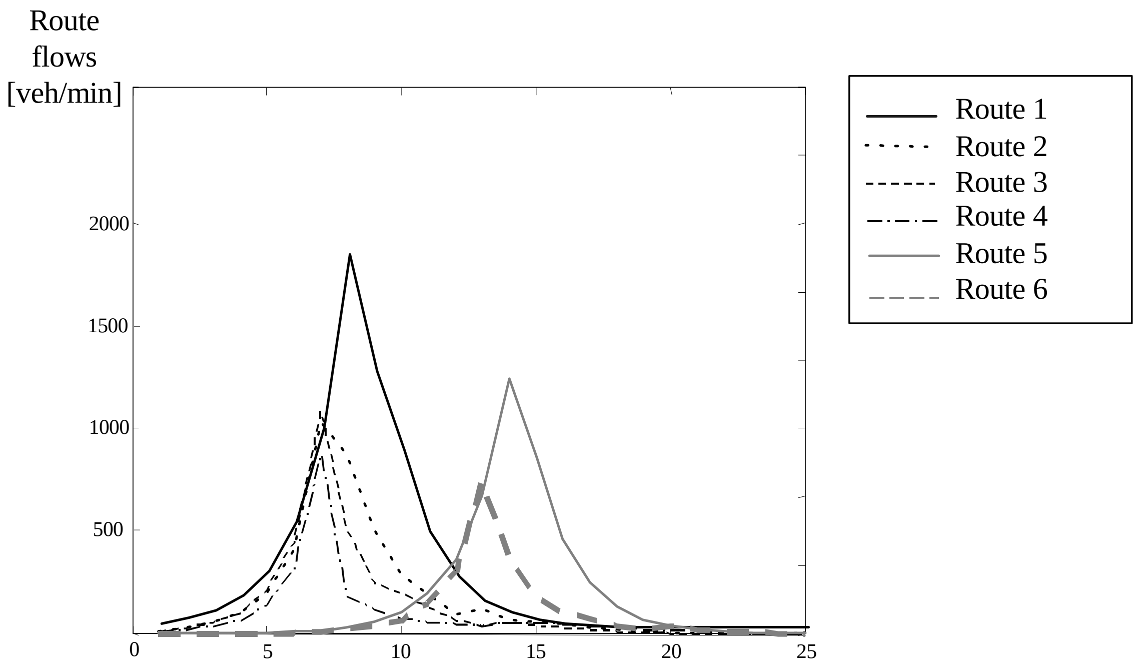

Figure 7.19: Route flows for zero toll case

On link 5 the following proportions will be applied:

$$\phi_5(t) = \left\{ \begin{array}{ll} 1.0, & \text{if } t = 17; \\ 0.8, & \text{if } t = 16, 18; \\ 0.6, & \text{if } t = 15, 19; \\ 0.4, & \text{if } t = 14, 20; \\ 0.2, & \text{if } t = 13, 21; \\ 0, & \text{otherwise.} \end{array} \right\} \tag{7.11}$$

This means that the optimal combination of toll values $\{\overline{\theta}_2, \overline{\theta}_5\}$ has to be determined for period 10 of link 2 and period 17 for link 5.

7.6.6 Results with tolls on links 2 and 5

Revenue maximization $E11$

Assume that the road authority aims to maximize toll revenues by selecting best pair of toll values on both tolled links (2 and 5). We applied grid search to find this optimum. For each pair of toll values, the dynamic traffic assignment (DTA) is solved. In Figure 7.20a, the total revenues are plotted for each toll value ($0 \leq \overline{\theta}_2 \leq 30$ and $0 \leq \overline{\theta}_5 \leq 30$). In the case toll levels on both links are zero, there are clearly no revenues. In contrast, for very high toll levels on both links, most travelers will choose to travel on the untolled routes, resulting in lower total toll revenues. As can be observed from Figure 7.20b tolling

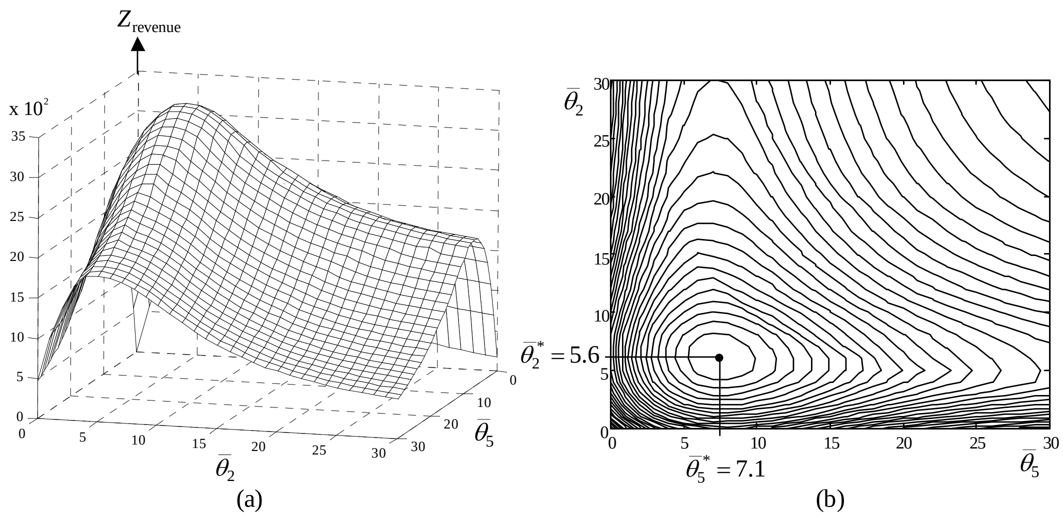

Figure 7.20: Results from experiment E11 Total toll revenues objective: a) revenue curve for different toll values, b) contour plot with optimal toll values

with $\overline{\theta}_2(t) = 5.6$ and $\overline{\theta}_5(t) = 7.1$ yields the highest revenues. As expected, link 5 will contribute significantly more to the revenues than link 2.

The resulting optimal toll pattern on link 2 is:

$$\theta_2(t) = \left\{ \begin{array}{ll} 5.6, & \text{if } t = 10; \\ 4.2, & \text{if } t = 9, 11; \\ 2.8, & \text{if } t = 8, 12; \\ 1.4, & \text{if } t = 7, 13; \\ 0, & \text{otherwise.} \end{array} \right\} \tag{7.12}$$

while the optimal toll pattern for link 5 is as follows:

$$\theta_5(t) = \left\{ \begin{array}{ll} 7.1, & \text{if } t = 17; \\ 5.7, & \text{if } t = 16, 18; \\ 4.3, & \text{if } t = 15, 19; \\ 2.8, & \text{if } t = 14, 20; \\ 1.4, & \text{if } t = 13, 21; \\ 0, & \text{otherwise.} \end{array} \right\} \tag{7.13}$$

The resulting route flows are depicted in Figure 7.21. Compared with the case of zero tolls in Figure 7.19, it can be seen that OD-1 travelers shift towards (non-congested and untolled) route 1 and also shift their departure times. In the non-toll case (Figure 7.19) there were about 250 travelers per minute on (untolled) route 1 at time period 5 while in the tolled case (Figure 7.21) there are about 500 travelers per minute at the same time period. If we consider time period 8, the situation is different. While in non-tolled case,

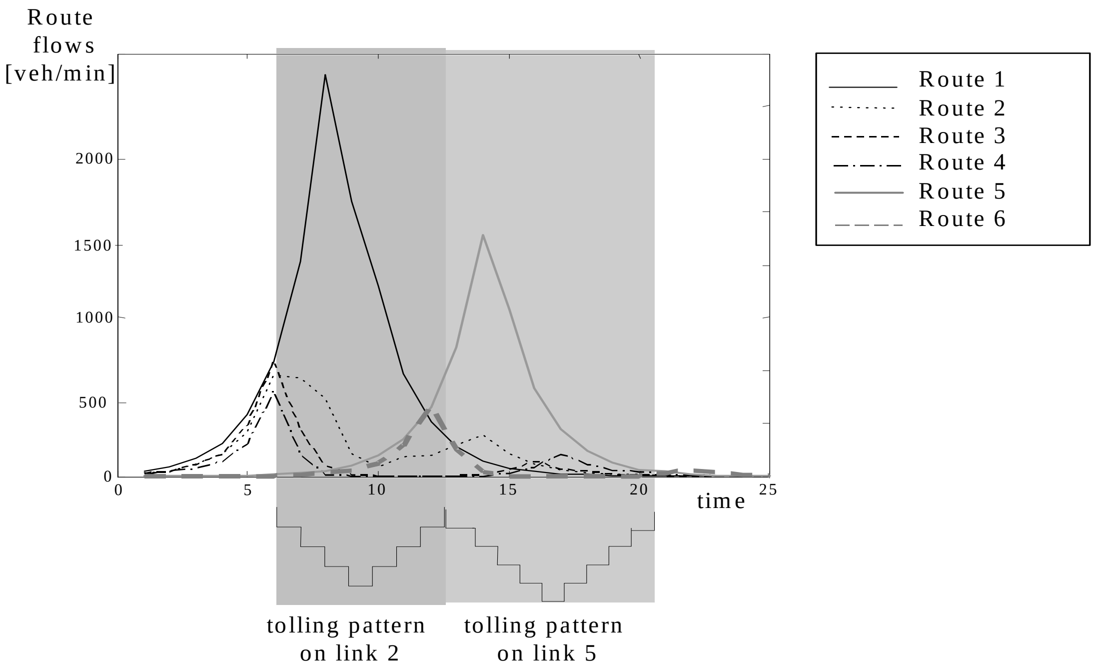

Figure 7.21: Route flows for the objective of maximizing toll revenues

about 1800 travelers were using route 1, in the tolled situation there are about 3000 travelers. The situation on (tolled) route 2 is opposite. In time period 8 in the non-toll case there were about 1000 travelers per minute while in the tolled case only 500 travelers per minute appear. Similarly, OD 2 travelers shift towards non-congested and non-tolled route 5 and avoid tolled route 6.

Total travel time minimization $E12$

In this case the road authority is assumed to aim at minimizing total travel time on the Chen network by selecting the best toll levels on both tolled links. Figure 7.22 depicts the total travel time for different toll levels for links 2 and 5. Again, upper and lower toll boundaries for links 2 and 5 are: $0 \leq \overline{\theta}_2 \leq 30$ and $0 \leq \overline{\theta}_5 \leq 30$. As can be observed from Figure 7.22, toll values $\overline{\theta}_2(t) = 6.1$ [eur/passage] and $\overline{\theta}_5(t) = 16.6$ [eur/passage] yield lowest travel time.

The resulting tolling pattern for link 2 is:

$$\theta_2(t) = \left\{ \begin{array}{ll} 6.1, & \text{if } t = 10; \\ 4.6, & \text{if } t = 9, 11; \\ 3.05, & \text{if } t = 8, 12; \\ 1.5, & \text{if } t = 7, 13; \\ 0, & \text{otherwise.} \end{array} \right\} \tag{7.14}$$

while the resulting toll pattern for link 5 is:

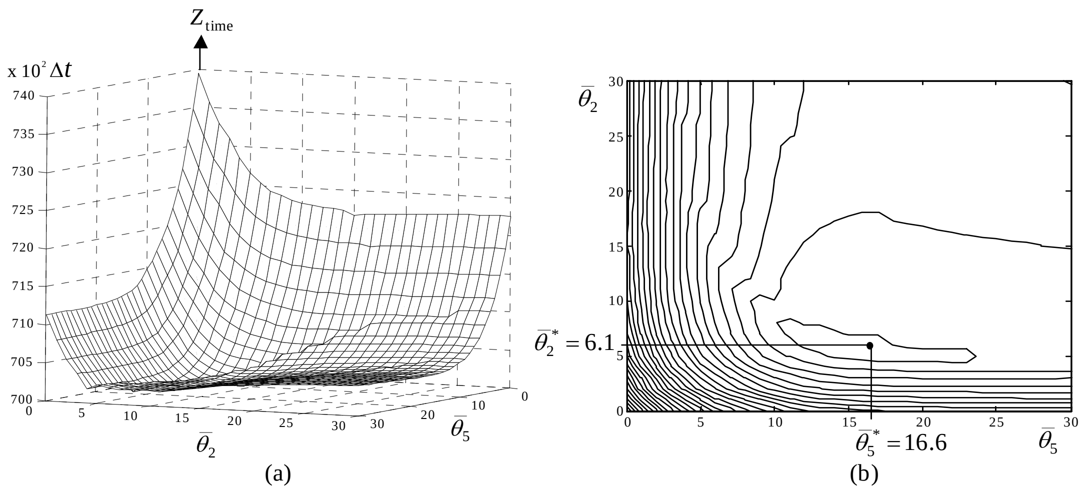

Figure 7.22: Results from experiment $E12$ Total travel times for different toll values: a) curve b) contour plot with optimal toll values

Table 7.14: Discussion of the results on Chen network

Objective: revenue generation			**Objective: travel time**		
optimal tolls	link 2	link 5	optimal tolls	link 2	link 5
	5.6	7.1		6.1	16.6
total revenues	33, 45x10^2		total travel time	700, 1x$10^2\Delta t$	
total travel time	706.06x$10^2\Delta t$		total revenues	26, 43x10^2	

$$\theta_5(t) = \left\{ \begin{array}{ll} 16.6, & \text{if } t = 17; \\ 13.2, & \text{if } t = 16, 18; \\ 9.9, & \text{if } t = 15, 19; \\ 6.6, & \text{if } t = 14, 20; \\ 3.3, & \text{if } t = 13, 21; \\ 0, & \text{otherwise.} \end{array} \right\} \tag{7.15}$$

It can be seen that with setting tolls on these links, the total travel time will decrease from about 740 x $10^2\Delta t$ [min] to 700 x $10^2\Delta t$ [min]. Travel times may decrease for some travelers if tolls are imposed because of spreading travel demand more evenly over less congested time periods and routes. On the other hand travelers travel times will increase due to a shift to longer routes in order to escape from the tolls.

7.6.7 Discussion of results

The results of the DOTD problem for both policy objectives for the multi-OD Chen network are outlined on Table 7.14.

The results show that with tolling of multiple links of the Chen network, the given policy objectives can be optimized. It is shown that the objective of revenue generation can be maximized ($Z_{revenues} = 33.45\text{x}10^2$ [eur]) if the tolls are equal to $\theta_2(t) = 5.6$ and $\theta_5(t) = 7.1$ for links 2 and 5, respectively. However, with these toll values the total travel time of $706.06\text{x}10^2\Delta t$ [min] is reached.

As can be seen from Table 7.14, the total travel time on the Chen network can be minimized ($Z_{time} = 700.10\ \text{x}10^2\Delta t$ [min]) with optimal tolls $\theta_2(t) = 6.1$ and $\theta_5(t) = 16.6$ on links 2 and 5, respectively. The revenue generation is however 26.43 x10^2 [eur] only.

In these case studies is shown that with tolls travelers are willing to shift to less tolled time periods and non-tolled routes in order to avoid to pay tolls. This appears for both OD pairs. An another study, by Liu & McDonald (1999), considers an urban highway network consisting of two routes connecting an origin and a destination. One route is tolled, and the other is untolled. Travelers' behavior (route and departure choice) is considered. The results show that the optimum second-best tolls have the following impacts on the allocation of traffic volume: a) diversion of peak period traffic to the free route, b) shifts of peak period traffic to the off-peak period; May & Milne (2000) tested and compared various kinds of road pricing scheme.

In this case study two (congested) links are tolled. However, different combinations of links are possible.

7.7 Summary and conclusions from experiments

In this chapter different case studies were presented in order to demonstrate the plausibility of the proposed model to solve the DOTD problem. We use the mathematical formulation of the DOTD problem (Chapter 5) as well as formulation of the dynamic traffic assignment model extended for road pricing (Chapter 6) and solve the DOTD problem. Different case studies are performed from different perspectives: network specifications, travel behavior, policy objectives, user class specifications, and tolling schemes. With each case study we aim to illustrate a specific aspect of the DOTD problem (explained in Section 7.3). The following case studies were performed with different purposes:

1. a small corridor network, where the departure time model is demonstrated. Its purpose is to show how user class specifications (differences in value of time and value of schedule delay) influence timing decisions. Thus, the impact of tolls given heterogeneity of users with regard to VOT and VOSD on temporal flow pattern is demonstrated; in Ubbels (2006) is shown that people with higher VOT and VOSD seem to find pricing more acceptable than people with lower VOT ans VOSD.

2. a dual route transportation network, where different tolling schemes are applied, namely, uniform, (quasi)uniform and variable tolling schemes. Its aim is to show how these different tolling schemes perform with regard to different policy objective

functions; Another study, by Liu & McDonald (1999), considers an urban highway network consisting of two routes connecting an origin and a destination. One route is tolled, and the other is untolled. Travelers' behavior (route and departure choice) is considered. The results show that the optimum second-best tolls the following impacts: a) diversion of peak period the free route, b) shifts of peak periods to the off-peak periods; May & Milne (2000) tested and compared various kinds of road pricing scheme.

3. finally, a multi-OD transportation network is taken into account where two different links are tolled. In this case the optimal toll pattern for both links is determined in order to satisfy given policy objectives. In Ubbels (2006) is shown that, for commuting trips, the changes in departure times are the most realistic to happen.

The belief that travelers will change not only routes (if any) but also departure time led us to including the corridor case in this experiments set-up. In order to give insight into changing departure time patterns of travelers after tolls are imposed. All case studies show that travelers will not only change routes but also their departure times in order to pay less for a trip (depending on their travel preferences). The resulting time-space flow pattern is not trivial and results from a complex interaction of different user types.

In most studies it is assumed that travelers have the same behavior and decide about a trip in the same way. Our aim was to show the impact of tolls given heterogeneity of travelers. Since different classes of travelers will react differently on tolls and change their travel behavior according to their travel preferences. The user class differentiation in this work is made with regard sensitivities for value of time as well as value of schedule delay. It is shown that the travelers with lower VOT and VOSD tend to change their routes and departure times more likely than travelers with higher VOT and VOSD. These finding are in line with Ubbels (2006).

Different tolling schemes are applied and analyzed on different transportation networks. Namely, it is shown that uniform, quasi(uniform) and variable tolling scheme give different results with regard to travel behavior as well as achieved policy objectives.

From the policy objective point of view, we investigated the effect of different tolling schemes on different policy objectives. Different policy objectives are analyzed, namely maximization of revenues and travel time minimization. These two objectives are chosen (among many possible) to illustrate constrasting impacts of policy objectives. It is shown that some tolling regimes are more appropriate to apply in order to achieve a prespecified policy objective. For example, for minimization total travel times, the variable tolling scheme seems more appropriate than a uniform scheme because of the possibility of switching to time periods. It is opposite for maximization of revenues. Clearly, the uniform tolling scheme performs best (simply, the travelers have no departure time choice).

The experiments are conducted on different types of transportation networks. While the simplest corridor network learn us more about the properties of the model, a multiple

OD-pair network as a more complex and realistic case is applied showing that the DOTD problem can be solved by tolling different links on the network. The overlapping of routes are considered in this experiment. In the next chapter (Chapter 8) the conclusions and further recommendations are given.

Chapter 8

Conclusions and Further Research

The subject of this thesis is the application of dynamic road pricing in dynamic networks. Both forms of dynamics represent the outstanding elements of this dissertation. Its objective is the formulation and testing of a design methodology for an optimized tolling system for road networks.

This chapter summarizes the research developed and presented throughout the thesis. Its scope and main contributions to the current state-of-the-art in dynamic road pricing methodology are briefly described in Section 8.1. A summary of work done in this thesis is given in Section 8.2. The new theoretical insights into the multi-actor dynamic road pricing formulation are shown in Section 8.3, in which different games are applied to the dynamic optimal toll design problem. The importance of taking into account differences of travel preferences for different groups of travelers is described in Section 8.4. In section 8.5, the conclusions with regard to policy objectives of the road authority are given. The importance of computing optimal toll design problem in dynamic (flow-dependent) fashion and the possibilities to extend developed model in various ways motivates further research, as indicated in Section 8.5. Finally, Section 8.6 concludes this thesis with some recommendations.

8.1 Scope of conducted research

Direct pricing of trips, for example using tolls, is widely advocated to solve problems in transportation planning such as congestion, environmental impacts, safety and the like. In many countries some form of road pricing is already functioning, be it as a mean to control the level of demand for car trips, to regulate the use of scarce capacity during peak hours, or to charge the road users for the cost of using new infrastructure.

An elaboration of crucial elements of the dynamic road pricing design problem for the development of a design optimization methodology is given in this thesis (Chapter 2).

Special attention in this thesis is given to differences in travel behavior of different types of travelers in contrast to most studies in travel behavior modeling considering homogenous travelers only. We believe that especially in the road pricing problem, the heterogeneity of travelers plays a crucial role. Hence our aim in this thesis was to consider explicitly differences in travel behavior and take these into account in the various modeling approaches.

In this thesis we took the position of a road authority trying to improve the performance of the transport system by adopting dynamic forms of road pricing. In order to assess the effectiveness of policy plans with respect to pricing, quantitative analysis tools (models) are needed that predict the impacts of road pricing. Moreover, such models may derive the best tolling pattern to be applied given a specific planning objective of the road authority. As a result, a toll design model is able to determine the optimal combination of characteristics of a toll regime (where, when, from whom and how much toll to levy). The behavioral aspects of the road pricing design problem are taken into account where behavioral responses of the travelers to the incurred tolls (shifts in trip frequency, route choice, departure time choice) are analyzed.

The required toll system design tool should have the following characteristics:

1. the toll design system tool can address dynamic networks, meaning that travel demand, network flows, travel times, capacities and the like may vary over time;
2. the toll design system tool can handle a heterogeneous composition of the travel demand with respect to drivers to be tolled differently and with respect to different responses to pricing;
3. the toll levels are dynamic in the sense of time varying.

The objectives of this research are briefly summarized as follows:

1. give insight into the impacts of road pricing in dynamic transportation networks;
2. establish a methodology suitable for solving the dynamic optimal toll design problem;
3. develop models for road pricing, (and modify existing travel behavior models as well as network loading models);
4. formulate and solve an optimization road pricing problem to determine dynamic optimal tolls;
5. demonstrate the correctness and soundness of proposed models on a set of experimental networks.

The main result of this thesis is the development and application of a design tool (including the developed travel choice models) for optimizing the system set-up (locations, periods, levels of tolls to be levied, etc) of different road pricing regimes in dynamic networks. Starting from the multi-user dynamic traffic assignment methodology developed and applied by Bliemer (2001), new formulations for different types of pricing, and various extensions needed to capture road pricing are developed. Moreover, in this thesis we extended a number of notions with respect to tolling to the situation of dynamic tolls to be applied in dynamic networks with dynamic demand. This refers to possible objectives, road pricing regimes, and road pricing measures. Namely, developing of the road pricing model (including different pricing regimes and different policy objectives) add an additional optimization level, which makes the formulation and solution of the dynamic optimal toll design problem more complex. The developed methodology is tested by applying on (small) dynamic networks, showing that the optimal toll settings can be determined if regimes are given and the travel demand is known.

8.2 Summary of conducted research

We started with presenting a description of the planning context and the multi-actor setting of road pricing as far as relevant for this thesis (Chapter 1). A classification and elaboration has then been given of the various types of tolling regimes considering all their dimensions (Chapter 2). Determining the best tolling regime given an authorities' objective is identified as a network design problem in which particular properties (fares, tolls etc) of links and maybe nodes of a network need to be determined.

After having introduced the tolling design problem, a microscopic demand analysis of tolling using a game theoretic approach has been conducted assuming utility maximizing behavior of travelers and of network operators (Chapter 3). To that end, descriptions have been given of different game types (Monopoly, Cournot, Stackelberg) suitable for microscopic tolling analysis. Based on a mathematical formulation of the different game types in case of tolling, an in depth quantitative analysis has been performed to show the differential outcomes with respect to optimal tolls given the assumed market power of involved actors, different objectives of road authorities, and heterogeneous composition of the traveler population (Chapter 4). Small experiments were performed with trip choice and route choice as possible response strategies of travelers to the incurred tolls.

After the microscopic approach a switch has been made to a macroscopic approach of the tolling design problem. In that respect a mathematical formulation has been established of the dynamic optimal tolling design (DOTD) problem given an objective of the road authority and assuming a set of possible travel choice responses (trip choice, route choice departure time choice) on the part of the road users (Chapter 5). The characteristics of this tolling design problem have been identified, one of which is that this problem belongs to the class of MPEC problems, Mathematical Programming with Equilibrium Constraints. An important part of this problem is solving the dynamic equilibrium of flows in the network consistent with the imposed tolls. We have formulated a stochastic dynamic equilibrium flow problem for the case of dynamic tolls, giving special attention to the formulation of the departure time choice model as most important travel choice dimension in this context (Chapter 6). The route and departure time choice models have been included as stochastic models. Similar as in our game theoretic approach, the heterogeneity of travelers is considered also in this macroscopic formulation of the dynamic optimal toll design problem. The heterogeneity is considered with respect to differences in value of time and value of schedule delay of travelers in the travel cost function.

The thesis concludes with a series of experiments and case studies aimed at showing the feasibility of the developed framework, and demonstrating the type of impacts of tolling on travel patterns and network conditions (Chapter 7). In total 12 different small scale experiments have been carried out on different network types, with different policy objectives, tolling types, and user class differentiation. In each experiment, the optimal dynamic toll pattern has been determined given these different conditions on the supply and demand sides.

8.3 Findings and Conclusions

The tolling system design problem has been identified as a network design problem, implying that the determination of optimal locations, optimal times and optimal levels of tolls is not straightforward but is highly dependent on the complex interactions between travelers in a network. It has also been shown that apart from multiple possible objectives a tolling system design has very many design dimensions (fare base, locations, periods, levels, user differentiation, fare collection, revenue use, etc.) that all affect response behavior of road users.

The in-depth microscopic analyses of toll pricing using the game theory approach have shown significant differences in outcomes resulting from different assumptions on the interactions among involved actors (authority versus travelers) as expressed in different game types adopted (Monopoly, Cournot, Stackelberg).

Depending on the assumed market power of the road authority and travelers, different optimal toll design games can be proposed. Taking all game concepts into account, the two-stage optimal toll design game seems to be most realistic because we expect that the road authority will influence the travel behavior of travelers and have more ‘market power’ than travelers.

In addition, this analysis already clearly shows that different objectives on the part of the road authority may lead to different tolling solutions.

It is shown that different user-classes lead to different outcomes of the games, both in terms of optimal toll as well as in payoffs for both the travelers and the road authority. However, due to among other matters assumed pure strategies and non-continuous cost functions, there may exist multiple optimal strategies (multiple Nash equilibrium solutions). It is shown that Stackelberg game performs best among other considered games (Monopoly and Cournot game). This microscopic analysis already reveals an important feature of the tolling design problem, namely that solutions (strategies, designs) maybe non-unique: multiple Nash equilibria may exist posing special requirements on solution approaches.

Although the game theoretic approach seems preferable from a theoretical and behavioral point of view, its practical implementation in large-scale networks seems not yet feasible due to its computational complexities.

While a microscopic approach helps to understand the responses to pricing of individual actors, for more realistic situations (e.g. large numbers of travelers), an aggregate level analysis has been followed. The nature of pricing (existence of different levels in decision making) suggests that the dynamic optimal toll design problem is a bi-level problem. Moreover, the nature of the lower level of the dynamic optimal toll design problem as a dynamic equilibrium problem leads to a Mathematical Program with Equilibrium Constraints (MPEC) formulation.

The importance of as much as possible realistic modeling of travel behavior (using dynamic travel cost functions with road pricing) has led us to stochastic multi-user model formulations.

From the model formulation it appeared a non-continuous travel cost function that combined with discrete tolling periods may lead to non-smooth objective functions requiring very specific solution procedures. The multi-user class dynamic network flow problem with dynamic tolls has been formulated as a variational inequality problem.

The experiments carried out with different small-scale networks gave much added value to the theoretical analyses. Indeed, the experiments revealed highly discontinuous cost functions requiring the adoption of a comprehensive grid search solution approach to finding the optimal toll levels. The experiments confirmed the high dependence of the tolling solutions to the adopted objectives. Their outcomes also corroborated the importance of user class distinction leading to highly different outcomes compared to the homogeneous case. The experiments impressively showed the impacts of the dynamic tolls on the temporal and spatial redistribution of trips in the networks.

Because of differences in value of time (VOT) and value of schedule delays (VOSD), time sensitive travelers (those with high VOT and VOSD) are willing to pay for an less congested trip than the cost sensitive travelers (these with low VOT and VOSD). Namely, the travelers with lower VOSD accept to travel outside of their preferred departure and arrival times because their penalties for departing earlier and arriving later are lower than those of travelers with higher values of schedule delays.

In this thesis, different policy objectives that the road authority can impose are studied. Only a few policy objectives have been analyzed. It should be noted that the proposed modeling framework, can be easily extended to include other policy objectives for the optimal toll design problem.

It is shown in our hypothetical cases that it seems possible to decrease total travel time in the network or maximize toll revenues by applying tolls. Moreover, there is an optimal pattern of tolls that optimizes the given objective function from the road authority. The impact of tolls on policy objectives where different groups of travelers have different values of schedule delay and value of time is analyzed. Interestingly, in comparing the objective functions (in our experiments), one may observe that not only the optimal toll values are similar but also that their corresponding objective function values are fairly close. Both adopted objectives (toll revenue maximization and travel time minimization) appear sensitive to small toll values.

8.4 Recommendations

Recommendations for further model developments

In the development of the models presented in this thesis a number of assumptions had to be made. Further research may release these assumptions in the following directions:

- To consider elastic demand (trip choice)

In our work we considered trip choice only in game theory approach, while in the macroscopic approach the trip choice of travelers was omitted (because of considering mandatory trips only). It can be nice improvement to consider elastic demand also in macroscopic approach and consider optional trips, too.

- Modifications of departure time choice model

Departure time choice seems to be crucial in modeling of road pricing. A more sophisticated departure time model can be used making the modeling of travel behavior more realistic. E.g. in this thesis we used the same values for parameters for deviation from preferred departure time (without difference between earlier or later departing). For more information see Van Amelsfort et al. (2005a).

- Tolling regime (flow dependent)

In this thesis the tolling is considered to be time varying, but with fixed proportions over time (Chapters 2 and 7). A more realistic approach of course might be to apply dynamic tolls where the toll values in each time period directly depend on the flow pattern.

- Long term behavioral changes

  In this thesis, only short term behavioral changes (route and departure time choice) are studied. The long term behavioral changes (for example, reallocation of firms or residence of travelers) after imposing of road pricing are the subject of the parallel study in the MD-PIT project (TIllema (2006)). It will be interesting to include these long term behavioral effects in the existing framework.

- Different policy objectives

Only a subset of possible policy objectives is studied in this thesis. Many more are possible. We developed however a generic model where easily other policy objectives can be applied. The functioning of the model is not influenced by changes of the objective function.

- Solution algorithm applied

In this thesis, it was not our focus to developed new solution algorithms for the dynamic optimal toll design problem. Given the non-continuous cost function, we applied a simple grid search algorithm (which seems to be satisfying for small hypothetical networks). However, much more sophisticated solution procedures can be developed to solve this problem. For example, heuristic methods (genetic algorithms) seem to be promising for this kind of problems.

- Dynamic travel cost function

The dynamic travel cost function can be further developed and improved in various ways. For example, the dynamic travel function can be easily extended to include the reliability aspect of traveling.

- Recommendations for model calibration/validation/applications

The models described in this thesis have a strong theoretical background, but are not yet applied in practice. The parameter values are taken from recent travel behavior studies while the outcomes of the model are checked for plausibility and soundness. However, the soundness of the model approaches presented in this thesis may improve considerably if comparison with real-life data is done.

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Summary

Road pricing is a controversial topic gaining high attention from society, not only researchers but planners, politicians, economists, travelers, etc. Pricing is generally seen as a promising approach to help solving today's transport problems such as congestion, funding of road projects, negative impact of traffic to the environment, etc. Although promising, designing and applying a pricing strategy is a complex issue. Looking at this problem from only one perspective seems not enough to solve the problem because many actors are involved. To support the road authority in optimizing the traffic network performance, a practical tool is needed. A combination of traffic modeling, mathematical models, and control is suitable to give support to traffic engineers in network optimization.

Special attention in this thesis is given to differences in travel behavior of different types of travelers, in contrast to most road pricing studies considering homogeneous travelers only. We believe that, especially in the road pricing problem, the behavioral differences between travelers play a crucial role. Hence, our aim in this thesis is to consider explicitly differences in travel behavior and take these into account in the various modeling approaches.

The main result of this thesis is the development and application of a design tool (including the developed travel choice models) for optimizing the system set-up (levels of tolls to be levied) of different road pricing regimes in dynamic traffic networks. Moreover, in this thesis we extended a number of notions with respect to tolling to the situation of time-varying tolls to be applied in dynamic traffic networks with dynamic travel demand. This refers to possible policy objectives, road pricing regimes, and road pricing measures. Namely, developing the road pricing model (including different pricing regimes and different policy objectives) adds an additional optimization level which makes the formulation and solution of the dynamic optimal toll design problem more complex. The developed methodology is assessed by applications on small test networks showing that the optimal toll settings can be determined if regimes are given and the travel demand is known.

The tolling system design problem has been identified as a network design problem, implying that the determination of optimal locations, optimal departure times and optimal levels of tolls is not straightforward but highly dependent on the complex interactions between travelers in a network. It has also been shown that apart from multiple possible

objectives, a tolling system design has many design dimensions (fare levels, user differentiation, fare collection, revenue use, etc.) that all affect response behavior of road users.

The in-depth microscopic analyses of toll pricing using the game theory approach have shown significant differences in outcomes resulting from different assumptions on the interactions among involved actors (authority versus travelers) as expected in different game types adopted. Depending on the assumed market power of the road authority and travelers, different optimal toll design games can be proposed. Taking all game concepts into account, the two-stage Stackelberg game seems to be most realistic because we expect that the road authority will influence the travel behavior of travelers and have more 'market power' than travelers. This analysis shows that different objectives on the part of the road authority may lead to different optimal tolling solutions.

While a microscopic approach helps to understand the responses to pricing of individual actors, for more realistic situations (e.g. large numbers of travelers), an aggregate level analysis has been followed. The nature of pricing (existence of different levels of decision making) suggests that the dynamic optimal toll design problem is a bi-level problem. The nature of the lower level of the dynamic optimal toll design problem as a dynamic equilibrium problem leads to a Mathematical Program with Equilibrium Constraints (MPEC) formulation. The importance of as much as possible realistic modeling of travel behavior (using dynamic travel cost functions with road pricing) has led us to a probabilistic multi-user model formulations. From the model formulation it appeared that a continuous travel cost function combined with discrete tolling periods may lead to non-smooth objective functions requiring specific solution procedures. The multi-user class dynamic network flow problem with dynamic tolls has been formulated as a variational inequality problem.

The experiments carried out with different small-scale networks gave much added values to the theoretical analyses. The experiments revealed highly discontinuous cost functions requiring the adoption of a comprehensive grid search solution approach to finding the optimal toll levels. The experiments confirmed the high dependence of the tolling solutions to the adopted objectives. Their outcomes also corroborated importance of user class distinction leading to highly different outcomes compared to the homogeneous case. The experiments impressively showed the impacts of the dynamic tolls on the temporal and spatial redistribution of trips in the hypothetical networks.

Because of differences in value of time and value of schedule delays, time sensitive travelers are willing to pay for a less congested trip than the cost sensitive travelers. Namely the travelers with lower value of schedule delay accept to travel outside of their preferred departure and arrival times because their penalties for departing earlier and arriving later are lower that those of travelers with higher values of schedule delays.

In this thesis, different policy objectives that the road authority can impose are studied. Only a few specific policy objectives are analyzed. It should be noted that the proposed modeling framework can be easily extended to include other policy objectives for the optimal toll design problem.

It is shown in our hypothetical cases that it seems possible to decrease total travel time in the network or maximize toll revenues by applying tolls. Moreover, there is an optimal pattern of tolls that optimizes the given objective function from the road authority. The impact of tolls on policy objectives where different groups of travelers have different values of schedule delay and value of time is analyzed.

Sadrzaj

Problem putarine predstavlja vrlo kompleksan problem koji zaokuplja paznju drustva u celini (ne samo naucnih radnika vec planera, politicara, ekonomista, korisnika puteva, itd.). Placanje putarine prihvaceno je kao potencijalno resenje transportnih problema u danasnjem drustvu, kao sto su saobracajne guzve, finansiranje izgradnje novih i odrzavanja postojecih puteva, negativan uticaj saobracaja na prirodno okruzenje, itd. Mada obecavajuce, projektovanje i primena mera putarine predstavlja vrlo kontraverzno resenje. Resavanje ovog kompleksnog problema u kome je ukljuceno vise nivoa odlucivanja i razmatranje problema putarine samo iz jedne perspective je nedovljno za stvarno resavanje ovog problema. Kao podrska drzavnim organima za upravu putevima pri optimizaciji saobracajne putne mreze, potreban je *paket mera* koji se moze testirati u praksi. Kombinacija modeliranja saobracajnih situacija, specijalnih matematickih modela i sistema za kontolu predstavlja pogodno resenje kao podrska saobracajnim inzenjerima pri optimizaciji saobracajne mreze.

U ovoj tezi posebna paznja je posvecena uticaju putarine na razlicite klase ucesnika u saobracaju (putnika) pri izboru ruta u saobracajnoj mrezi. Ovaj aspekt analize odlucivanja putnika je posebnost u odnosu na vecinu studija koje se bave ovom tematikom i koje pretpostavljaju jednoobrazno saobracajno ponasanje i odlucivanje svih ucesnika. Autori veruju da, narocito u problemu odredjivanja optimalne visine putarina, razlike u odlucivanju razlicitih klasa vozaca igraju kljucnu ulogu. Zbog toga, cilj autora je da jasno i nedvosmisleno obuhvate razlike u odlucivanju vozaca (pri izboru ruta, vremena polaska, itd) i da ih uzmu u obzir pri izradi razlicitih varijanti nacina modeliranja putarine.

Najznacajniji rezultat ove teze je razvijanje i primena projektnog modela za optimizaciju parametara sistema (lokacije primene putarine, vremenskih perioda, visine putarine koje ce biti propisane, itd.) razlicitih rezima putarine u dinamickim saobracajnim mrezama. Stavise, u ovoj tezi autori su prosirili dosadasnji aspekt modelovanja putarine za razlicite situacije dinamicke putarine u dinamickim saobracajnim mrezama, ciljeve organa za upravu putevima u vezi sa putarinom, i mogucih mera primene putarine. Medutim, razvijanje modela putarine (ukljucujuci razlicite rezime putarine i razlicite ciljeve organa za upravu putevima) zahteva dodatni nivo optimizacije. Zbog toga je formulisanje i optimizacija celokupnog problema odredjivanja optimalne dinamicke putarine u dinamickim saobracajnim mrezama dosta slozeno. Razvijena metodologija je testirana na malim saobracajnim mrezama, pokazujuci da optimalne dinamicke putarine mogu biti odredjene ako su poznati ciljevi primene putarine i ukupan broj putnika na saobracajnoj mrezi.

Problem odredjivanja dinamicke putarine u dinamickoj saobracajnoj mrezi pripada klasi problema projektovanja saobracajnih mreza, gde odredjivanje optimalnih lokacija putarine, optimalnih polazaka putnika sa odredista, i optimalnih nivoa putarine zavisi od slozenih interakcija izmedju putnika u saobracajnoj mrezi i karakteristika same mreze. Pokazano je da problem dinamicke putarine ima razlicite dimenzije (visina putarine, lokacije, klasa putnika, nacin prikupljanaja putarine, koriscenje novca prikupljenog putarinom) ima veliki uticaj na saobracajne odluke putnika (koju rutu iskoristiti, kada krenuti na put ili da li put uopste preduzimati).

Detaljna i sveobuhvatna analiza problema odredjivanja optimalne putarine primenom teorije igara pokazala je znacajne razlike u rezultatima. Zavisno od trzisnog uticaja organa za upravu puteva sa jedne strane, i putnika sa druge strane, definisane su razlicite vrste postavki igara sa putarinom. Uzimajuci vise razlicitih vrsta igara u obzir, takozvana igra u dve faze (Stackelber igra) je najrealija. U ovoj vrsti igre, pretpostavlja se da organi za upravu putevima uticu na saobracajne odluke putnika u vecoj meri nego obratno (postavljanjem putarina, odredjivanjem visine putarine, itd). Drugim recima, uprava puteva ima vise 'trzicnog uticaja' na putnike. Analiza razlicitih igara sa putarinom pokazuje da razliciti ciljevi organa za upravu puteva vode do razlicitih (nekada suprotnih) predloga za resavanje problema optimalne putarine.

Dok ovaj pristup dozvoljava razmatranje efekata putarine individualnih ucesnika u igri, za realnije situacije (na primer, veci broj putnika i vece saobracajne mreze), drugaciji prilaz je neophodan. Sama priroda procesa putarine (a posebno postojanje razlicitih nivoa odlucivanja) asocira da problem odredjivanja dinamicke putarine pripada klasi takozvanih bi-level (dvo-slojnih) problema. Priroda nizeg nivoa problema odredjivanje optimalne dinamicke putarine (takozvanog problema ekvilibrijuma saobracajne mreze), dovela je to postavke problema u obliku MPEC-a (Matematickog programa sa ogranicenjem u ekvilibrijumu). Vaznost modelovanja saobracajnog odlucivanja putnika dovela je do razvijanja stohasticke postavke ovog problema sa razlicitim klasama (vrstama) ucesnika (vozaca). Postavka ovakovog modela za odredivanje optimalne dinamicke putarine se odlikuje diskontnualnom funkcijom. Dalje, ova funkcija kombinovana sa diskretnim peridima primene putarine dovodi do veoma kompleksne ciljne funkcije, zahtevajuci specijalne matematicke i optimizacione metode za resavanje ovog problema.

Experimenti nad razlicitim saoobracajnim mrezama (manjeg formata) dali su znacaj teorijskim postavkama ovog problema. Zbog kompeksnosti funkcije koju treba optimizovati, metoda pretazivanja 'grid search' je koriscena za odredjivanje optimalnih vrednosti dinamicke putarine. Eksperimenti pokazuju visoku zavisnost resenja dinamicke putarine od primenjene ciljne funkcije koju propisuje uprava za puteve (visi nivo problema). Takodje, razlike izmedju klasa putnika (u odnosu na njihove demografski- socijalne karakteristike kao sto su godine starosti, prihod, pol, zanimanje, itd.) vode do velikih razlika u odnosu na slucaj kada se svi putici ponasaju na isti nacin i imaju iste karakteristike. Experimenti jasno pokazuju uticaj dinamicke putarine na vremenski i prostorni raspored pojedinacnih putovanja u saobracajnom mrezi.

Zbog razlika u vrednovanju vremena i odstupanja od planiranog vremena dolaska ili polaska na put, pojedini putnici (oni vise zavisni od vremena) su spremni da plate visu putarinu za razliku od putnika koji nisu vremenski ograniceni. Putnici sa nizim vrednostima vremena, su spremni da radije prihvate putovanje van planiranog polaznog vremena i vremena stizanja na odrediste, a da bi izbegli placanje putarine. To je objasnjeno time da ovi putnici nisu vremenski ograniceni ali su zato osetljiviji na cenu putarine. U ovoj tezi, razlicite ciljne funcije organa za upravu putevima su analizirane (na primer, smanjiti ukupno vreme putovanja na saobracajnoj mrezi ili povecanje ubiranja putarine). Samo neke od mogucih opcija su uzete u obzir. Treba naznaciti da ovaj pristup moze veoma lako biti prosiren za bilo koju drugu ciljnu funkciju organa za upravu putevima.

Pokazano je da je moguce smanjiti ukupno vreme putovanja u saobracajnoj mrezi ili povecati ubiranje putarine primenom optimalne dinamicke putarine. Stavise, postoji optimalna visina putarine koja dovodi do optimizacije datih ciljeva. Uticaj putarine na ciljeve ubiranja putarine analiziran je za slucaj razlicitih klasa putnika.

About the author

Dusica Joksimovic was born on 8th of June 1971 in Belgrade, Serbia. In 1995 she received a master's degree (with distinction) in Information Systems at the University of Belgrade, Faculty of Technology, Management and Information Systems. The master's thesis concerned building of advanced graphical user interfaces using object oriented methodology. During her study Dusica was sponsored (as outstanding student) from the Serbian Ministry of Education.

After graduation in 1995, Dusica started as junior consultant to work for the highest goverment transportation institution -Transport Research Institute (CIP) in Belgrade. The interests focus, among others, on various transportation projects: designing a railway information system, systems for maintenance of railways, cost and benefit analysis, asset management, etc.

She also worked in the private company 'Oracle- Parallel' as a member of Oracle Corporation in designing of a database for railways for former Yugoslavian countries using Oracle database systems. She worked for Transport Research Institute in Belgrade until 2001 when she moved to the Netherlands. She worked first at Faculty of Mathematics at Delft University of Technology, on developing Java applets for the students exercises and on application of artificial intelligence.

Since 2002, Dusica is affiliated with the Transport and Planning Section of the Faculty of Civil Engineering and Geosciences, where she has been involved in her PhD research "Bi-level optimal toll design approach for dynamic networks" which is a part of the NWO research programme 'Multidisciplinary Study - Pricing in Transport'.

From October 2005 she works as a consultant in the transportation company ARS &Traffic and Technology where she is involved, among others, in the Spitsmijden project, where pricing experiments are carried out in practice in The Netherlands.

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